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Maritime Data Transmission Coverage Optimization Under Power and Distance Constraints

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ABSTRACT

A problem of the minimum-energy broadcast routing is considered for efficient maritime data transmission coverage. The power emitted by a ship radio station is limited, and this limitation tethers distance. Given an initial number of ships and their locations, they are triangulated. Upon the triangulation, the edges exceeding the maximum edge length equivalent to the maximum distance are removed. If the resulting graph has no disconnected ships, the solution is the minimum spanning tree. For two or more disconnected subgraphs, a minimum spanning tree is built for each of them, and the corresponding set of the efficient solutions is formed. Within this set, no minimum spanning tree exists that would either be shorter by connecting no fewer than a number of ships from an efficient solution or be not longer by connecting more than a number of ships from an efficient solution. The respective optimization problem, consisting in minimizing the broadcasting route length along with maximizing the number of ships communicating through the route, is solved by scalarizing the two criteria. The scalarization consists in standardizing the two criteria and calculating the distance of every achievable standardized efficient solution to the unachievable standardized solution. The percentage of two or more disconnected subgraphs is about 60%, whereas it is about 80% probable that the number of efficient solutions is equal to the number of disconnected subgraphs.

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1 Introduction

Maritime industry is one of the broadest industries related to and based on waterborne transportation, domain safety maintenance, mining and recovery operations. Ships are atomic components of the maritime industry, and they have to intercommunicate within a flotilla system and outside to ensure safe and effective functioning [1, 2]. The maritime communication is implemented via wireless data transmission systems [3, 4, 5].

Maritime data transmission includes, but is not limited to, technical information, possible route chart changes, obstacles, weather conditions and forecast, duty rearrangement, and, surely, broadcasting distress alerting along with search-and-rescue operations coordination [6, 7, 8]. Shipping is fulfilled as along seasides, maritime boundaries, as well as across midsea. Maritime traffic is very intense near seasides, so seaside broadcast-communication networks must be available and operate reliably [2, 4, 6, 9, 10].

Worldwide automated emergency signal communication for ships at sea is managed by the Global Maritime Distress and Safety System (GMDSS) [9, 11, 12]. This system is supplemental to the International Convention on Maritime Search and Rescue adopted in 1979 and provides basis for the communication [11, 13]. The basis includes safety procedures, types of equipment, and communication protocols used for safety and rescue operations of the distressed ships, boats, and other vessels. The GMDSS consists of several systems which are intended to perform alerting ships in the vicinity and ashore authorities [13], search and rescue coordination [14], locating (homing), maritime safety information broadcasts [4, 5], general communications (part of which is mentioned above), and bridge-to-bridge communications [15]. The GMDSS is used along with Maritime Safety & Security Information System (MSSIS), which is an unclassified, near real-time data collection and distribution network [16]. The MSSIS combines shared data from Automatic Identification System (AIS) [4, 5, 8, 17], coastal radars [10], and other maritime broadcasters [16] into a single data stream through secure Internet-based servers [5]. The AIS itself allows ships viewing marine traffic in their area and to be seen by that traffic. Through the MSSIS, participating governments can view real-time AIS data from all over the world in various geographic visualizations [16, 18].

To maintain such dense and high-quality data exchange, maritime data transmission coverage should be kept constantly rationalized. For instance, an AIS transceiver operating within very high frequency range transmits at 2, 5, or maximum 12.5 watts, and this limits the ship visibility to about 37 kilometers [3, 11, 19]. The visibility range is further stretched over radio stations of ships in the vicinity [20, 21]. In maritime data transmission coverage, therefore, the power limitation tethers distance. Another distance limitation emerges from the need of speeding up search and rescue operations [6, 7, 13, 14]. While a ship is in distress, it needs a rescue crew to get to it as fast as possible. Thus, there is no need to broadcast distress alerting too far for the sake of preserving battery supply, as well as for not impeding other maritime operations (within regular maritime traffic) being fulfilled farther. In the first turn, the emergency information should be broadcasted to the nearest vessels [13, 14, 17, 21].

2 Motivation and goal

A ship radio station located on a maritime vessel emits electromagnetic energy whose power is limited by the ITU Radio Regulations [22] and additional requirements established by law or international treaty [1, 2]. Larger cargo ships traveling the open seas may be required to have long-distance communications equipment, whereas coastal ships may only need short range communications [3, 9, 10, 23]. In a search-and-rescue operation, it is vitally important that the maritime vessels participating in it (including the vessel towards which the search-and-rescue operation must be accomplished) be communicated reliably using reasonably their power resources.

On the one hand, grand total power emitted by the vessels around the search-and-rescue area is limited to comply with electromagnetic compatibility requirements [24, 25]. On the other hand, a maritime broadcast-communication network should approximate the minimum-energy broadcast routing to ensure efficient coordination within the network [4, 7, 14, 20]. The ship locations are basically provided by the AIS either through the MSSIS or via short-distance radio communications. Knowing the locations, the minimum-energy broadcast routing can be projected by the method of building the Euclidean minimum spanning tree [26, 27]. The tree connects a given number of nodes (herein, also referred to as ships) without any cycles and passing through the minimum possible total distance [28, 29]. An issue may emerge from availability of multiple minimum spanning trees, which cover different numbers of ships by different routes. The goal is to find the best possible minimum-energy broadcast routing. For achieving the goal, the following seven tasks are to be fulfilled:

1. To describe how power and distance limitations (or constraints, in terms of mathematical modeling) affect the projection of a maritime broadcast-communication network.
2. To formalize the constraints in approximating the minimum-energy broadcast routing.
3. To formalize an objective to be optimized by minimizing the broadcasting route length along with maximizing the number of ships communicating through the route.
4. To suggest a method to efficiently solve the optimization problem.
5. To obtain statistics on how the method performs.
6. Based on the statistics, to discuss the practical applicability and significance of the suggested method.
7. Based on the results obtained and impartially discussed, to conclude on the contribution to the field of maritime data transmission coverage. A way of possible extension of the research will be outlined.

3 Power and distance constraints

Denote a number of ships located within a maritime domain by N. It is assumed that the broadcast-communication network cannot emit too much power in order to ensure electromagnetic compatibility. This means that each ship transceiver is limited to a maximum power it can emit. Consequently, the distance between two ships at which they can communicate is limited also. Denote this longest distance by d_{max}. An example of the network for N = 10 ships is presented in Figure 1, where the longest distance is shown as well in the same scale. When the constraint of the longest distance is applied, there are 23 ship pairs (out of 45 ship pairs)

{1, 2}, {1, 3}, {1, 4}, {2, 6}, {2, 7}, {2, 8}, {2, 10}, {3, 4},
{3, 5}, {3, 6}, {3, 7}, {3, 8}, {3, 9}, {3, 10}, {4, 5}, {4, 6},
{4, 7}, {4, 8}, {4, 10}, {5, 8}, {7, 8}, {8, 9}, {8, 10},

which appear too distant to communicate directly. The communication between ships in these pairs can be established via other ships. For instance, ships 1 and 2 can communicate via ship 9 being a proxy for them. Meanwhile, ships 3 and 10 can communicate via proxy ships 2, 9, 5. The two other communication paths {2, 9, 6} and {2, 9, 7} are longer (that is visually seen) and therefore are not efficient.

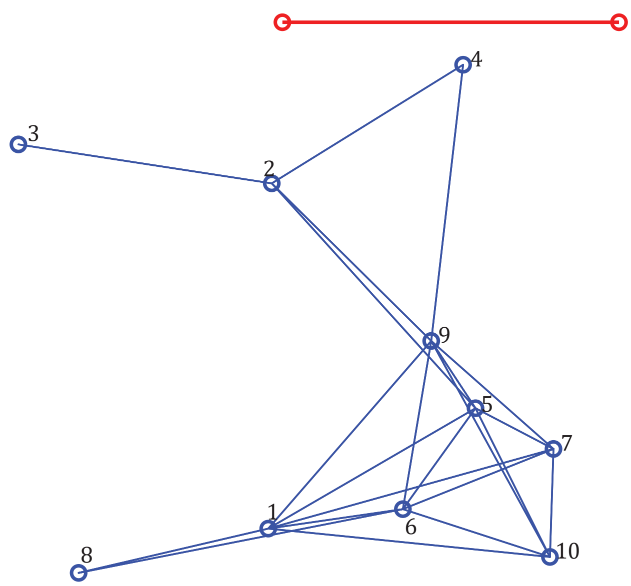

Figure 1 A version of the broadcast-communication network for an instance of 10 ships; the constraint of the longest distance shown above is applied and 23 ship pairs appear too distant to communicate directly

Source: Author

Although the direct communication between ships 8 and 6 in this example is possible, it would be less efficient than connecting these ships via proxy ship 1. The similar situation is noticeable for ship pairs {2, 5}, {9, 10}, {1, 7}. Less obvious are proxies for ships pairs {9, 7}, {1, 10}. However, this is just a still of the network; the ships are moving and those local situations constantly change.

Sliver triangles like those with ship-vertices {8, 1, 6}, {2, 9, 5}, {9, 5, 10}, {1, 6, 7} in Figure 1 are not efficient for the direct communication between the ships distanced the farthest. Therefore, along with removing the direct communication between ships distanced farther than by $d_{\max}$, the longest sides of sliver triangles should be removed also. This can be done by applying the Delaunay triangulation to the set of N ships, which are planar nodes over which the triangulation is performed [30, 31].

4 Data transmission coverage

Initially, the Delaunay triangulation is applied to N ships as N planar nodes

$$S = \{\mathbf{S}_i\}_{i=1}^{N} = \{[x_i \quad y_i]\}_{i=1}^{N}, \tag{1}$$

where x_i and y_i are conditionally horizontal and vertical coordinates, respectively, of ship $\boldsymbol{S}_i$ as if it is located on the Euclidean plane. The Euclidean distance between ships $\boldsymbol{S}_j$ and $\boldsymbol{S}_k$ is

$$\rho_{\mathbb{R}^2}(\mathbf{S}_j, \mathbf{S}_k) = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2} = d(\mathbf{S}_j, \mathbf{S}_k)$$

$$\text{by } j = \overline{1, N} \text{ and } k = \overline{1, N}. \tag{2}$$

The result of the triangulation is an undirected graph over N planar nodes (1) connected by a set of U edges

$$E = \{\mathbf{E}_u\}_{u=1}^{U} = \{[j_u \quad k_u]\}_{u=1}^{U}$$

$$\text{by } j_u \in \{\overline{1, N}\},\ k_u \in \{\overline{1, N}\},\ j_u \neq k_u, \tag{3}$$

where edge $\boldsymbol{E}_u$ connects ships $\boldsymbol{S}_{ju}$ and $\boldsymbol{S}_{ku}$. The length of edge $\boldsymbol{E}_u$ is $l(\mathbf{E}_u) = d(\mathbf{S}_{j_u}, \mathbf{S}_{k_u})$ for $u = \overline{1, U}$.

The edges whose length is greater than $d_{\max}$ are removed and thus an edge subset of set (3) is formed:

$$E^* = \{\mathbf{E}_{u_p}\}_{p=1}^{P} =$$

$$= \{\mathbf{E}_u : d(\mathbf{S}_{j_u}, \mathbf{S}_{k_u}) \leqslant d_{\max}, u = \overline{1, U}\} \subset E \text{ by } P \leqslant U. \tag{4}$$

Subset (4) consists of P edges whose length does not exceed $d_{\max}$. These P edges may connect N ships or fewer. The latter occurs when a ship is distanced farther than by $d_{\max}$ from each of the remaining $N - 1$ ships.

The objective is to connect a maximum possible number of ships by minimizing the broadcast-communication network length. The length minimization is realized by a minimum spanning tree connecting N^* ships via $N^* - 1$ edges

$$\overline{E}^* = \{\overline{\mathbf{E}}_n\}_{n=1}^{N^*-1} \subset E^* \subset E \tag{5}$$

from subset (4), where $N^* \leqslant N$, and N^* ships connected via $N^* - 1$ edges (5) constitute subset S^*. Therefore, the objective is to maximize number N^* over subset $S^* \subset S$ by minimizing the sum of the lengths of $N^* - 1$ edges (5) connecting all the ships in this subset:

$$N^{**} = \max_{N^* \in \{\overline{2, N}\}, S^* \subset S} N^* \text{ by } l^{**} = \min_{S^* \subset S} \sum_{n=1}^{N^*-1} l(\overline{\mathbf{E}}_n). \tag{6}$$

In fact, the maximization and minimization in (6) constitute a two-criterion problem.

It is quite clear that if $E^* = E$ then $P = U$, and the minimum of the broadcast-communication network length is achieved at a minimum spanning tree covering N nodes (1). Then problem (6) is solved as

$$N^{**} = N,\ l^{**} = \sum_{n=1}^{N-1} l(\overline{\mathbf{E}}_n). \tag{7}$$

In this case, where the maximum edge length $d_{\max}$ exceeds every edge length following the triangulation, the minimum spanning tree connecting the maximum of ships

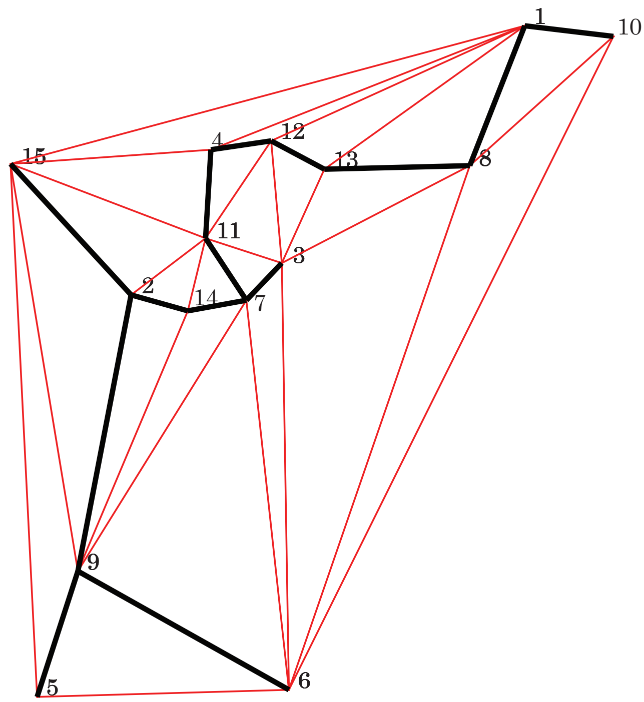

Figure 2 An example of 15 nodes representing 15 ships, which are not distanced too far from each other (the edge length is either not constrained or the maximum edge length constraint is sufficiently long); following the triangulation, no edges are removed from the set of U = 37 edges, and the minimum spanning tree (whose edges are highlighted bold) connecting all the 15 ships is the solution

Source: Author

ensures the minimum-energy broadcast routing (see an example in Figure 2). If $E^{*} \neq E$ then it means that at least one edge of set (3) has been removed, so $P < U$. However, if just one edge is removed, then $P = U - 1$, and N ships still can be connected via a minimum spanning tree covering N respective nodes (1). This case is shown in Figure 3 by the example from Figure 2, where the single removed edge is highlighted by dash line.

The solution tree can cover fewer than the initial number of nodes only if at least two edges are removed, i. e. by $P < U - 1$. The latter inequality, however, does not always imply smaller data transmission coverage. Thus, further shortening the longest possible distance to directly communicate for the example in Figure 3 does not change the solution for a certain interval of the constraint (Figure 4). Eventually the solution abruptly changes (Figure 5), wherein the three ships (5, 6, 9) are disconnected from the network (if one of the 12 remaining ships is selected as the hotspot – e. g., the ship that needs help and is broadcasting distress alerting). The coverage length in Figure 5 is 1.6447 times shorter than that in Figure 4, so "losing" three ships (out of 15 ships, which is 20%) seems admissible.

On the other hand, too severe maximum edge length constraints may result in that set (4) is broken into two or more disconnected sets. Figure 5 is an example of such disconnection – the three edges connecting ships 5, 6, 9 have fallen off due to the edges connecting ships 2 and 9, and ships 14 and 9 have been removed (the edge connecting ships 2 and 9 belongs to the minimum spanning tree in Figure 4). Although the coverage length in Figure 6 is 2.7192 times shorter than that in Figure 5, the number of connected ships is decreased fourfold.

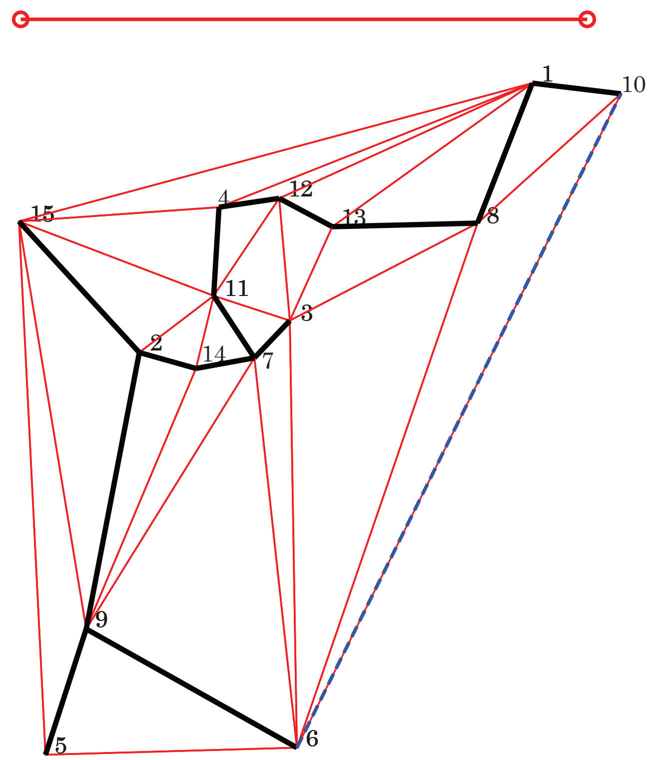

Figure 3 The example of 15 ships from Figure 2, where the longest possible distance to directly communicate is shown above; following the triangulation, only the edge connecting ships 6 and 10 is removed (the edge connecting ships 6 and 8 is slightly shorter than d_{max}), but the minimum spanning tree remains the same (Figure 2) connecting all the 15 ships

Source: Author

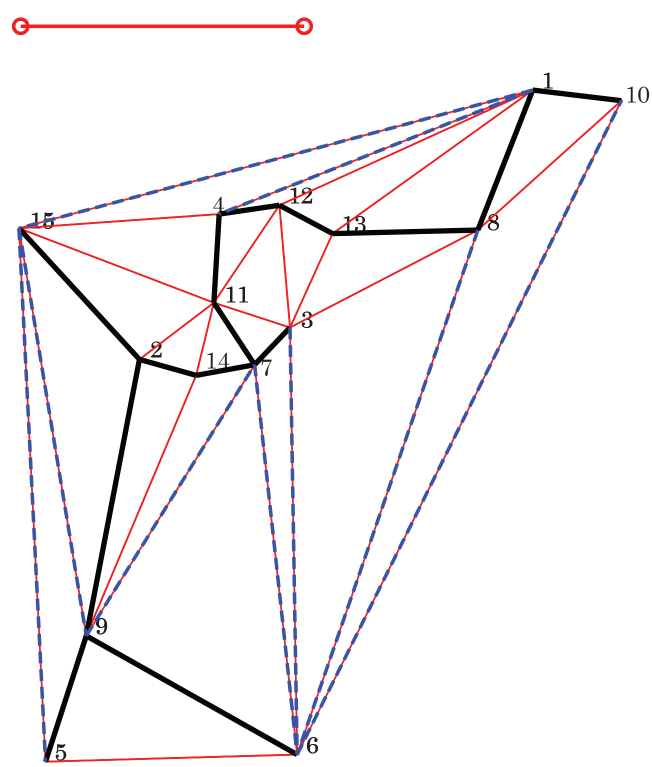

Figure 4 The example of 15 ships from Figure 2, where the maximum edge length constraint shown above is made severer, whereupon nine edges are removed; despite the removals, the minimum spanning tree remains the same (Figure 2) still connecting all the 15 ships

Source: Author

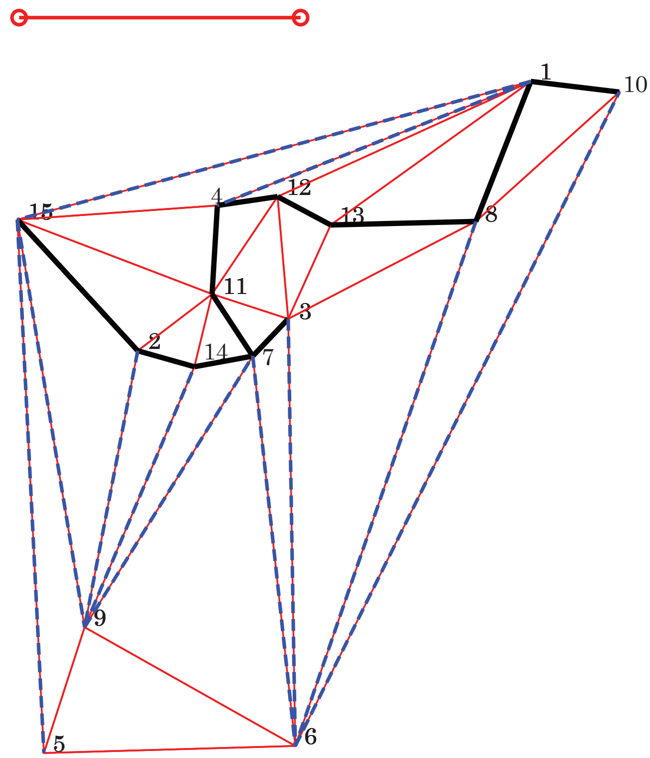

Figure 5 The example of 15 ships from Figure 4 upon making the maximum edge length is little shorter (the difference between this constraint and the one in Figure 4 is very small and hardly can be noticed), whereupon two more edges are removed (the edges connect ships 2 and 9, and ships 14 and 9); the consequence of the additional removals is a minimum spanning tree covering only 12 nodes

Source: Author

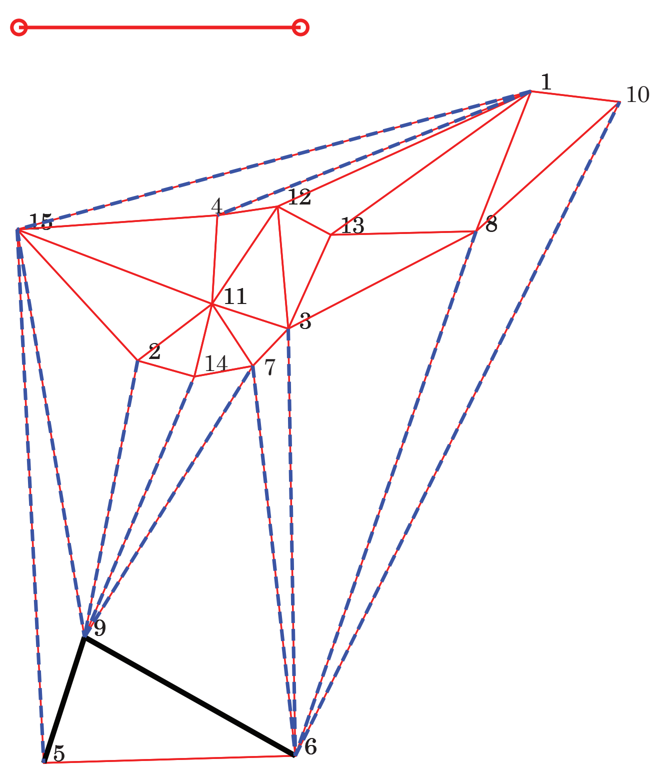

Figure 6 The example of 15 ships from Figure 5, where changing the hotspot ship to 5, 6, or 9 results in the other minimum spanning tree (nearly the most primitive tree) covering just those three nodes (the maximum edge length constraint is the same as it is in Figure 5)

Source: Author

If a graph with edges (4) contains fewer than N nodes, set (4) is a union of Q disconnected subsets $\left\{E_q^*\right\}_{q=1}^{Q}$:

$$E^* = \bigcup_{q=1}^{Q} E_q^* \text{ by } \bigcap_{q=1}^{Q} E_q^* = \varnothing \text{ and } Q \in \mathbb{N} \setminus \{1\}. \tag{8}$$

In fact, subsets $\left\{E_q^*\right\}_{q=1}^{Q}$ are disconnected subgraphs. Then there are Q versions of the minimum spanning tree, over which two-criterion problem (6) is supposed to be solved. Denote these Q versions of the solution by

$$\left\{N_q^{**}, l_q^{**}\right\}_{q=1}^{Q}. \tag{9}$$

Besides, denote the minimum spanning tree of solution $\left\{N_q^{**}, l_q^{**}\right\}$ by T_q. Further, only the efficient solutions are considered. If exists $q_0 \in \left\{\overline{1, Q}\right\}$ such that either

$$N_q^{**} \geqslant N_{q_0}^{**} \text{ and } l_q^{**} < l_{q_0}^{**} \tag{10}$$

or

$$N_q^{**} > N_{q_0}^{**} \text{ and } l_q^{**} \leqslant l_{q_0}^{**} \tag{11}$$

for $q \in \left\{\left\{\overline{1, Q}\right\} \setminus \{q_0\}\right\}$ then solution $\left\{N_{q_0}^{**}, l_{q_0}^{**}\right\}$ is inefficient. Indeed, when the pair of inequalities (10) holds, tree T_{q_0} is longer than tree T_q, while the latter covers no fewer nodes than tree T_{q_0} does. When the pair of inequalities (11) holds, tree T_{q_0} covers fewer nodes than tree T_q does, while tree T_{q_0} is not shorter than tree T_q.

Upon removing every inefficient solution from set (9), a subset

$$\left\{\widehat{N}_h^{**}, \breve{l}_h^{**}\right\}_{h=1}^{H} \subset \left\{N_q^{**}, l_q^{**}\right\}_{q=1}^{Q} \tag{12}$$

of H efficient solutions remains. In subset (12) no solution $\left\{\widehat{N}_{h^*}^{**}, \breve{l}_{h^*}^{**}\right\}$ exists such that a pair of inequalities

$$\widehat{N}_{h^*}^{**} \geqslant \widehat{N}_h^{**} \text{ and } \breve{l}_{h^*}^{**} < \breve{l}_h^{**} \tag{13}$$

holds, nor holds a pair of inequalities

$$\widehat{N}_{h^*}^{**} > \widehat{N}_h^{**} \text{ and } \breve{l}_{h^*}^{**} \leqslant \breve{l}_h^{**} \tag{14}$$

for every $h^* \in \left\{\overline{1, H}\right\}$ by $h^* \neq h$. That is inequalities (13) and (14) are impossible if $\left\{\widehat{N}_{h^*}^{**}, \breve{l}_{h^*}^{**}\right\}$ is an efficient solution. Consequently, it is convenient to sort the efficient solutions in ascending order so that $\widehat{N}_h^{**} \leqslant \widehat{N}_{h+1}^{**}$, $\breve{l}_h^{**} \leqslant \breve{l}_{h+1}^{**}$ $\forall h = \overline{1, H-1}$.

The best data transmission coverage (i. e., the most efficient coverage) must be among the set of those H efficient solutions (12) with respective trees $\left\{\breve{T}_h^{**}\right\}_{h=1}^{H}$ associated with the solutions. If $H = 1$ then tree $\breve{T}_1^{**}$ with its length $\breve{l}_1^{**}$ covering $\widehat{N}_1^{**}$ nodes is the solution to the two-criterion problem (6). If $H > 1$ then a solution must be selected among H efficient solutions (12), and this selection will be an additional problem. Problem (6) in this case, generally speaking, does not have an exact solution [32, 33]. Therefore, the best approximate solution should be selected to accomplish as close as possible the operations of maximization and minimization in problem (6).

5 Scalarization

To solve the two-criterion optimization problem (6) for the case of $H > 1$, it is the best to scalarize it [34, 35]. Before the scalarization, both the criteria should be standardized. Instead of solutions (12), solutions

$$\left\{\overline{\widehat{N}}_h^{**}, \overline{\breve{l}}_h^{**}\right\}_{h=1}^{H} \tag{15}$$

are considered, where

$$\overline{\widehat{N}}_h^{**} = \frac{\widehat{N}_h^{**}}{\max\limits_{k=\overline{1, H}} \widehat{N}_k^{**}} \tag{16}$$

and

$$\overline{\breve{l}}_h^{**} = \frac{\breve{l}_h^{**}}{\max\limits_{k=\overline{1, H}} \breve{l}_k^{**}} \tag{17}$$

for $h = \overline{1, H}$. Upon the standardization by (16) and (17), both the criteria do not exceed 1 by not dropping below a threshold determined by a minimum-to-maximum ratio. Thus, the first criterion domain of a maximum possible number of ships becomes an interval

$$\left[\frac{\min\limits_{k=\overline{1, H}} \widehat{N}_k^{**}}{\max\limits_{k=\overline{1, H}} \widehat{N}_k^{**}}; 1\right] \tag{18}$$

of value (16) by the threshold being the left endpoint. The second criterion domain of a minimum possible length of the broadcast-communication network becomes an interval

$$\left[\frac{\min\limits_{k=\overline{1, H}} \breve{l}_k^{**}}{\max\limits_{k=\overline{1, H}} \breve{l}_k^{**}}; 1\right] \tag{19}$$

of value (17) by the threshold being the left endpoint.

Number

$$\bar{\bar{N}}^{**}=\max_{h=\overline{1,H}}\bar{\bar{N}}_h^{**}=1 \tag{20}$$

is the best (maximum) value of the first criterion, and number

$$\bar{\bar{l}}^{**}=\min_{h=\overline{1,H}}\bar{\bar{l}}_h^{**}=\frac{\min_{k=\overline{1,H}}\breve{l}_k^{**}}{\max_{k=\overline{1,H}}\breve{l}_k^{**}} \tag{21}$$

being the threshold of the second criterion is the best (shortest) length of the broadcast-communication network. Standardized solution $\left\{1,\bar{\bar{l}}^{**}\right\}$ by (20), (21) is, generally speaking, unachievable. The distance of an achievable standardized solution $\left\{\bar{\bar{N}}_h^{**},\bar{\bar{l}}_h^{**}\right\}$ to the unachievable standardized solution $\left\{1,\bar{\bar{l}}^{**}\right\}$ is

$$v_h=\sqrt{\left(\bar{\bar{N}}^{**}-\bar{\bar{N}}_h^{**}\right)^2+\left(\bar{\bar{l}}_h^{**}-\bar{\bar{l}}^{**}\right)^2}=$$

$$=\sqrt{\left(1-\bar{\bar{N}}_h^{**}\right)^2+\left(\bar{\bar{l}}_h^{**}-\frac{\min_{k=\overline{1,H}}\breve{l}_k^{**}}{\max_{k=\overline{1,H}}\breve{l}_k^{**}}\right)^2}\ \text{ for } h=\overline{1,H}. \tag{22}$$

Value (22) can be considered a (conventional) score of an efficient solution $\left\{\widehat{N}_h^{**},\breve{l}_h^{**}\right\}$. Herein, by convention, better solutions have lesser scores. The number of the best achievable standardized solution is

$$h^*\in\left\{\arg\min_{h=\overline{1,H}}v_h\right\}\subset\left\{\overline{1,H}\right\} \tag{23}$$

and thus problem (6) is solved as

$$N^{**}=\widehat{N}_{h^*}^{**},\ l^{**}=\breve{l}_{h^*}^{**}. \tag{24}$$

Consider an example of 25 ships and a maximum edge length constraint whose application leads to five disconnected subgraphs (Figure 7), i. e. $Q = 5$, where 36 edges are removed from the initial set of 65 edges. Each subgraph has its minimum spanning tree, and these five solution versions are (sorted in ascending order):

$$N_1^{**}=2,\ l_1^{**}=2.8843, \tag{25}$$

$$N_2^{**}=2,\ l_2^{**}=18.1434, \tag{26}$$

$$N_3^{**}=4,\ l_3^{**}=34.849, \tag{27}$$

$$N_4^{**}=7,\ l_4^{**}=57.2381, \tag{28}$$

$$N_5^{**}=9,\ l_5^{**}=90.1698. \tag{29}$$

Solution (25) is of the one-edge tree connecting ships 21 and 23. The other one-edge tree solution (26), where ships 14 and 19 are connected, is inefficient due to its route is longer. Solutions (27) – (29) are efficient, and so is solution (25). Therefore, $H = 4$ and the four efficient solutions can be re-written within the indexing of subset (12):

$$\widehat{N}_1^{**}=2,\ \breve{l}_1^{**}=2.8843, \tag{30}$$

$$\widehat{N}_2^{**}=4,\ \breve{l}_2^{**}=34.849, \tag{31}$$

$$\widehat{N}_3^{**}=7,\ \breve{l}_3^{**}=57.2381, \tag{32}$$

$$\widehat{N}_4^{**}=9,\ \breve{l}_4^{**}=90.1698. \tag{33}$$

The unachievable standardized solution is

$$\left\{1,\bar{\bar{l}}^{**}\right\}=\left\{1,\frac{\min_{k=\overline{1,4}}\breve{l}_k^{**}}{\max_{k=\overline{1,4}}\breve{l}_k^{**}}\right\}=$$

$$=\left\{1,\frac{2.8843}{90.1698}\right\}=\{1,0.032\}. \tag{34}$$

Four distances (22) to (34) are:

$$v_1 = 0.55,\ v_2 = 0.466,\ v_3 = 0.4543,\ v_4 = 0.6845. \tag{35}$$

By using (23), solution (32) is the closest to (34), so the problem solution is

$$N^{**}=\widehat{N}_3^{**}=7,\ l^{**}=\breve{l}_3^{**}=57.2381. \tag{36}$$

The best tree shown in Figure 7 connects seven ships, but its score is not much less than the score of solution (31) whose tree would connect four ships 3, 24, 10, 8. Solution (33) whose tree would connect nine ships has the worst score due to the tree is too long.

In setting the efficient solution score by (22), it is presumed that both criteria have the same importance. However, what if one of them is preferred more? In such a case, which basically generalizes the scoring by (22), the distance of an achievable standardized solution $\left\{\bar{\bar{N}}_h^{**},\bar{\bar{l}}_h^{**}\right\}$ to the unachievable standardized solution $\left\{1,\bar{\bar{l}}^{**}\right\}$ is

$$v_h(\alpha)=\sqrt{\alpha\cdot\left(\bar{\bar{N}}^{**}-\bar{\bar{N}}_h^{**}\right)^2+(1-\alpha)\cdot\left(\bar{\bar{l}}_h^{**}-\bar{\bar{l}}^{**}\right)^2}=$$

$$=\sqrt{\alpha\cdot\left(1-\bar{\bar{N}}_h^{**}\right)^2+(1-\alpha)\cdot\left(\bar{\bar{l}}_h^{**}-\frac{\min_{k=\overline{1,H}}\breve{l}_k^{**}}{\max_{k=\overline{1,H}}\breve{l}_k^{**}}\right)^2}\ \text{ for } h=\overline{1,H} \tag{37}$$

and some α ∈ (0;1) [35, 36]. A greater value of α means a greater importance of maximizing the number of connected ships. If α > 0.5 then the maximum possible number of

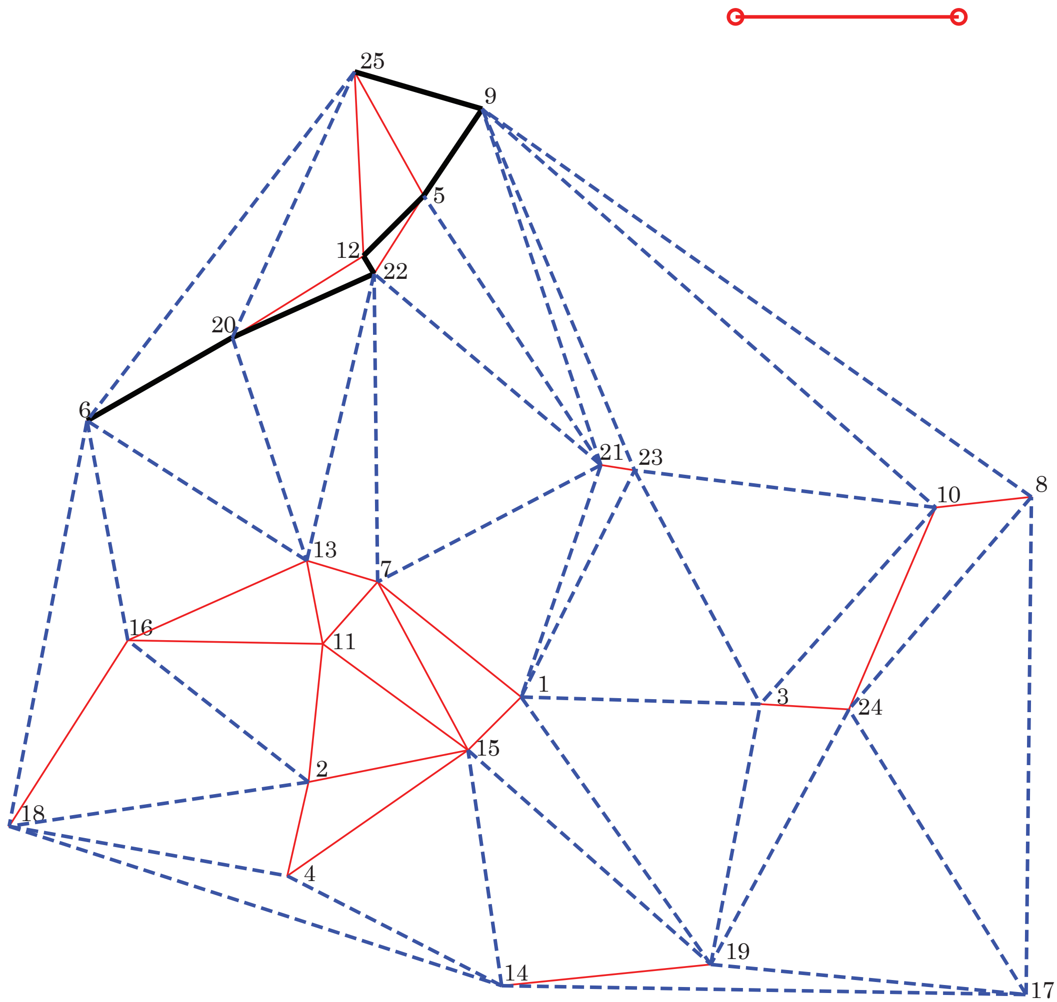

Figure 7 An example of 25 ships and a maximum edge length constraint whose application leads to five disconnected subgraphs (36 edges are removed from the set of U = 65 edges, so P = 29), four of which contain efficient solutions; the best achievable solution tree connecting seven ships is highlighted bold

Source: Author

connected ships is of a greater importance than the broadcast-communication network length minimum. Otherwise, if α < 0.5 then it is more important to minimize the broadcasting route length than maximizing the number of ships communicating through the route. The number of the best achievable standardized solution

$$h^* \in \left\{\arg\min_{h=\overline{1,H}} v_h(\alpha)\right\} \subset \left\{\overline{1,H}\right\} \tag{38}$$

depends on α, whereupon (24) is an eventual solution depending on α also. If α = 0.5 then both criteria have the same importance, just like by (22).

Suppose that maximizing the number of connected ships in the example of 25 ships in Figure 7 is more preferable. So, α should be set greater than 0.5. At α = 0.921 four distances (37) to (34) are:

$$v_1(0.921) = 0.7464,\ v_2(0.921) = 0.5424,$$
$$v_3(0.921) = 0.2724,\ v_4(0.921) = 0.2721. \tag{39}$$

By using (38), now solution (33) is the closest to (34), so the problem solution is

$$N^{**} = \widehat{N}_4^{**} = 9,\ l^{**} = \breve{l}_4^{**} = 90.1698 \tag{40}$$

covering the maximum possible number of nodes by the longest route (Figure 8). Nevertheless, not only the solution has changed. Having sorted distances (39) in ascending order, there is a preference chain of the four solutions that can be posted via trees $\left\{\breve{T}_h^{**}\right\}_{h=1}^{4}$. This chain is

$$\breve{T}_4^{**} \succ \breve{T}_3^{**} \succ \breve{T}_2^{**} \succ \breve{T}_1^{**}. \tag{41}$$

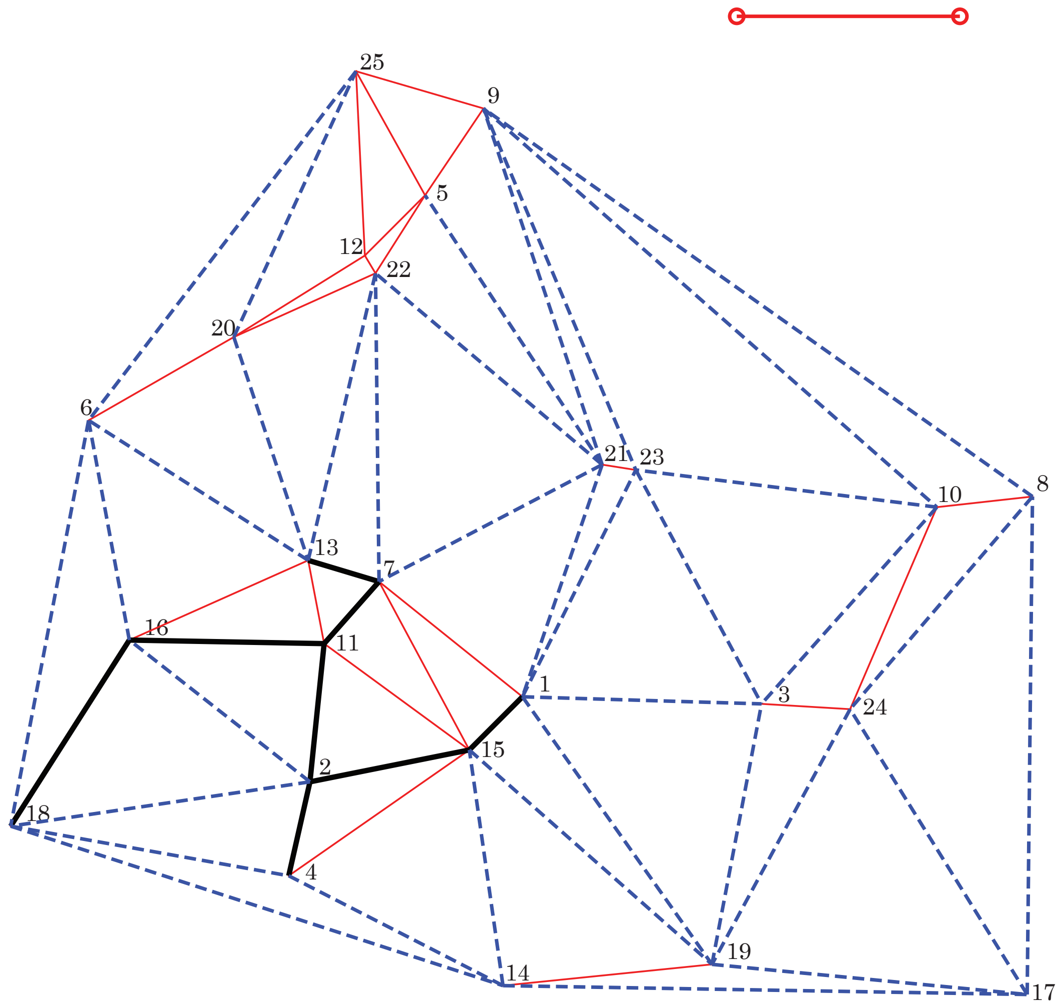

Figure 8 The example of 25 ships from Figure 7, where an additional preference to maximize the number of connected ships have been applied; the preference is so high that the solution changes despite the minimum spanning tree (highlighted bold) connected nine ships is 1.5753 times longer than the tree in Figure 7

Source: Author

Furthermore, preference chain (41) holds true for $\alpha \geqslant 0.921$ (to be more precise, it is $\alpha \geqslant 0.9208$, but it is a roundoff boundary, anyway), whereas it is

$$\breve{T}_3^{**} \succ \breve{T}_2^{**} \succ \breve{T}_1^{**} \succ \breve{T}_4^{**} \quad (42)$$

for $\alpha = 0.5$. Comparing chain (41) to (42), it is worth noting that the worst solution by equal importance of the criteria has become the best one by sufficiently increasing the importance of the number of ships to be connected. Besides, the remaining three solutions ($\breve{T}_3^{**}$, $\breve{T}_2^{**}$, $\breve{T}_1^{**}$) have not changed their preference interrelation.

In general, setting an additional preference to maximize the number of connected ships needs justification, unless it is $\alpha = 0.5$. In the particular example in Figures 7 and 8, setting $\alpha = 0.921$ has been intentional to show how the best solution, as well as the preference chain of the possible solutions, could be manipulated. In real-world situations, setting $\alpha > 0.5$ at some justified value does not guarantee that the solution by $\alpha = 0.5$ will change. The scores $\{v_h(\alpha)\}_{h=1}^{H}$ nonetheless do change, and pairwise ratios among them may be further used to substantiate the selection of α for an updated configuration of the ship locations (due to their constant change as the ships are moving).

6 Practical applicability and significance

To obtain statistics on how the scalarization method by (15) – (23) performs (for the sake of simplicity, α is set to 0.5), the number of initial nodes is varied between 10 and 50 with a step of 5:

$$N \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}. \quad (43)$$

Table 1 The average number of disconnected trees

	λ	0.75	0.8	0.85	0.9	0.95	1	1.05	1.1	1.15	1.2	1.25	Averaged over λ
N	10	1.1	1.19	1.25	1.27	1.34	1.48	1.68	1.79	1.89	1.9	1.97	1.5327
	15	1.1	1.17	1.3	1.37	1.45	1.64	1.83	2.12	2.29	2.43	2.6	1.7545
	20	**1.09**	1.12	1.23	1.46	1.77	1.91	2.18	2.35	2.78	2.96	3.27	2.0109
	25	1.14	1.16	1.27	1.39	1.6	1.9	2.2	2.68	3.01	3.22	3.5	2.0973
	30	1.1	1.23	1.33	1.62	1.74	1.95	2.34	2.78	3.2	3.57	3.93	2.2536
	35	1.13	1.27	1.4	1.66	1.8	2.01	2.37	2.93	3.33	3.86	4.3	2.3691
	40	1.23	1.3	1.55	1.76	2	2.22	2.71	3.36	3.81	4.52	5.08	2.6855
	45	1.2	1.3	1.56	1.76	2.05	2.49	2.94	3.41	4.15	4.46	5.1	2.7655
	50	1.25	1.31	1.52	1.71	2.03	2.67	3.15	3.85	4.29	4.89	**5.64**	2.9373
Averaged over *N*		1.1489	1.2278	1.3789	1.5556	1.7533	2.03	2.3778	2.8078	3.1944	3.5344	3.9322	**2.2674**

Source: Author

This makes up nine versions of the size of set (1). The maximum edge length is determined by the average edge length as

$$d_{\max} = \frac{1}{\lambda U} \cdot \sum_{u=1}^{U} l(\mathbf{E}_u) \tag{44}$$

by a constraint factor λ set between 0.75 and 1.25 with a step of 0.05:

$$\lambda \in \{0.75, 0.8, 0.85, 0.9, 0.95, 1, 1.05, 1.1, 1.15, 1.2, 1.25\}. \tag{45}$$

As the constraint factor is increased, the maximum edge length constraint becomes severer. Thus, (44) and (45) make up 11 versions of the constraint. For each pair {N, λ} the initial set of nodes (1) is generated as

$$\mathbf{S}_i = [x_i \quad y_i] = \\ = [80 \cdot \theta_i + 10 \cdot \vartheta_i + 50 \quad 80 \cdot \xi_i + 10 \cdot \zeta_i + 50] \text{ for } i = \overline{1, N} \tag{46}$$

for 100 times, where θ_i, ξ_i are values of two independent random variables distributed uniformly on the open interval (0;1) and ϑ_i, ζ_i are values of two independent random variables distributed normally with unit variance and zero mean [37, 38]. So, there are 9900 versions of the maritime data transmission coverage problem.

First, the matter of interest is how probable the situation is when the maximum edge length constraint application renders edge set (3) into edge set (4) connecting fewer than N ships. That is, how the number of disconnected trees (subgraphs) depends on the initial number of ships and the constraint severity. Table 1 presents the average number of disconnected trees (i. e., number Q averaged over 100 re-generations of the problem for particular N and λ). It is well-seen that as either N increases or λ is increased, or they both increase, the average number of disconnected trees increases. The average Q varies between 1.09 and 5.64 (highlighted bold). Averaged over N, it monotonously increases from 1.1489 at λ = 0.75 up to 3.9322 at λ = 1.25; averaged over λ, it monotonously increases from 1.5327 for 10 ships up to 2.9373 for 50 ships. The overall average number of disconnected trees is 2.2674 (highlighted bold).

Considering the 9900 optimization problems individually, number Q varies between 1 and 12. The case when set (4) includes all initial ships, i. e. when the minimum spanning tree is single and problem (6) is solved as (7), is very likely – it has occurred 3914 times (out of 9900), which is 39.5354%. The case with two disconnected trees is also likely – it has occurred 2639 times (26.6566%). Besides, there have been 1605 versions of set (4) with three disconnected trees (16.2121%). The cases with four, five, and six disconnected trees having occurred 882 (8.9091%), 468 (4.7273%), and 253 (2.5556%) times, respectively, are expectedly less likely. The case when the number of disconnected trees exceeds 6 is rare – its percentage is 1.404% (Figure 9).

Obviously, the average number of efficient solutions presented in Table 2 is less than the average number of disconnected trees. Similarly to the latter, the average number of efficient solutions increases as either N increases or λ is increased, or they both increase. The average H varies between 1.07 and 4.09 (highlighted bold). Averaged over N, it monotonously increases from 1.1389 at λ = 0.75 up to 3.0189 at λ = 1.25; averaged over λ, it monotonously increases from 1.4391 for 10 ships up to 2.4264 for 50 ships. The overall average number of efficient solutions is 1.9851 (highlighted bold) being just 12.4504% less than the overall average number of disconnected trees. This means that the share of inefficient solutions is rather

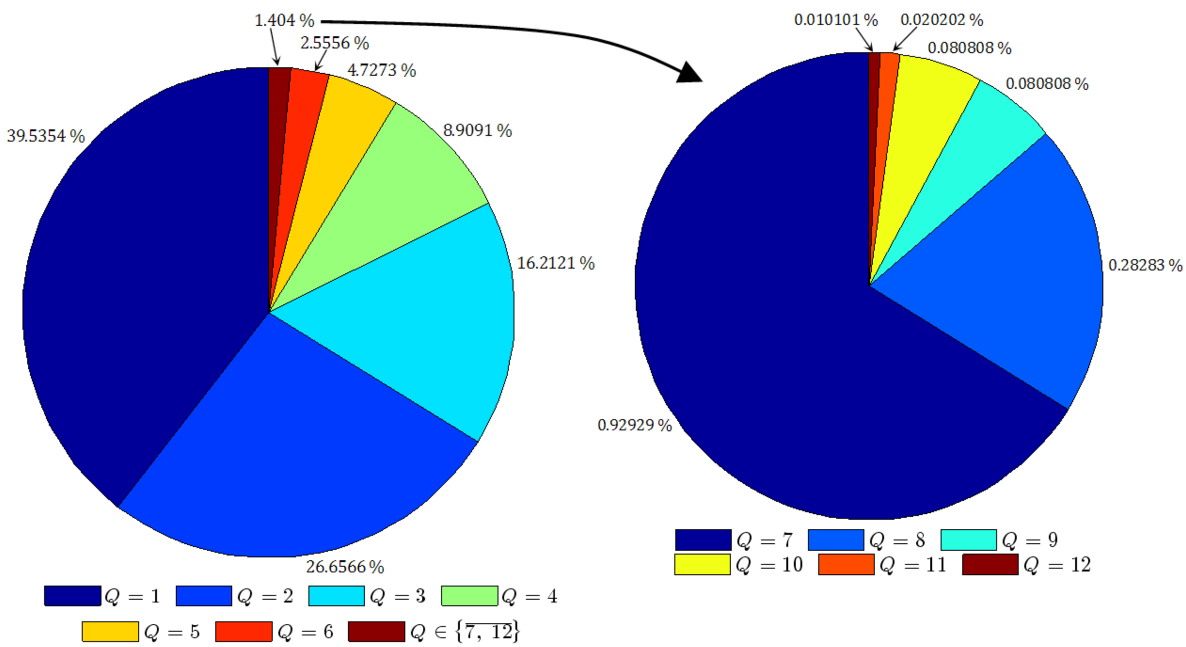

Figure 9 The pie chart of the number of disconnected trees

Source: Author

small, and application of the scalarization method by (15) – (23), or, in general, by (15) – (21), (37), (38), is quite necessary.

Number H varies between 1 and 7 considering the 9900 optimization problems individually. As the single minimum spanning tree ($Q = 1$) has occurred 3914 times, the case with a single efficient solution has occurred 3987 times (40.2727%), where 3914 optimization problems are those without tree disconnection and the other 73 problems have had tree disconnections ($Q \geqslant 2$). The optimization problem with two efficient solutions is very likely also – it has occurred 3197 times (32.2929%). So is the problem with three efficient solutions occurred exactly 1800 times (18.1818%). Four efficient solutions have occurred 734 times (7.4141%), whereas the optimization problem with five to seven efficient solutions is rare – its percentage is 1.8384% (Figure 10). There have been 158 problems with five efficient solutions, 23 problems with six efficient solutions, and only a one problem with seven efficient solutions. The latter is of 50 ships by the severest

Table 2 The average number of efficient solutions

	λ	0.75	0.8	0.85	0.9	0.95	1	1.05	1.1	1.15	1.2	1.25	Averaged over λ
N	10	**1.07**	1.16	1.21	1.25	1.31	1.39	1.58	1.67	1.72	1.71	1.76	1.4391
	15	1.09	1.16	1.29	1.33	1.39	1.55	1.76	1.94	2.08	2.14	2.25	1.6345
	20	1.09	1.12	1.21	1.43	1.66	1.78	1.98	2.11	2.4	2.47	2.58	1.8027
	25	1.14	1.16	1.25	1.38	1.54	1.8	2	2.32	2.64	2.72	2.81	1.8873
	30	1.1	1.23	1.33	1.55	1.62	1.83	2.1	2.46	2.69	2.85	2.97	1.9755
	35	1.13	1.27	1.38	1.63	1.74	1.9	2.19	2.55	2.89	3.24	3.37	2.1173
	40	1.23	1.3	1.49	1.69	1.89	2.03	2.33	2.77	3.07	3.33	3.6	2.2482
	45	1.19	1.29	1.51	1.65	1.9	2.24	2.59	2.91	3.25	3.41	3.74	2.3345
	50	1.21	1.27	1.48	1.67	1.97	2.44	2.71	3.01	3.2	3.64	**4.09**	2.4264
Averaged over N		1.1389	1.2178	1.35	1.5089	1.6689	1.8844	2.1378	2.4156	2.66	2.8344	3.0189	**1.9851**

Source: Author

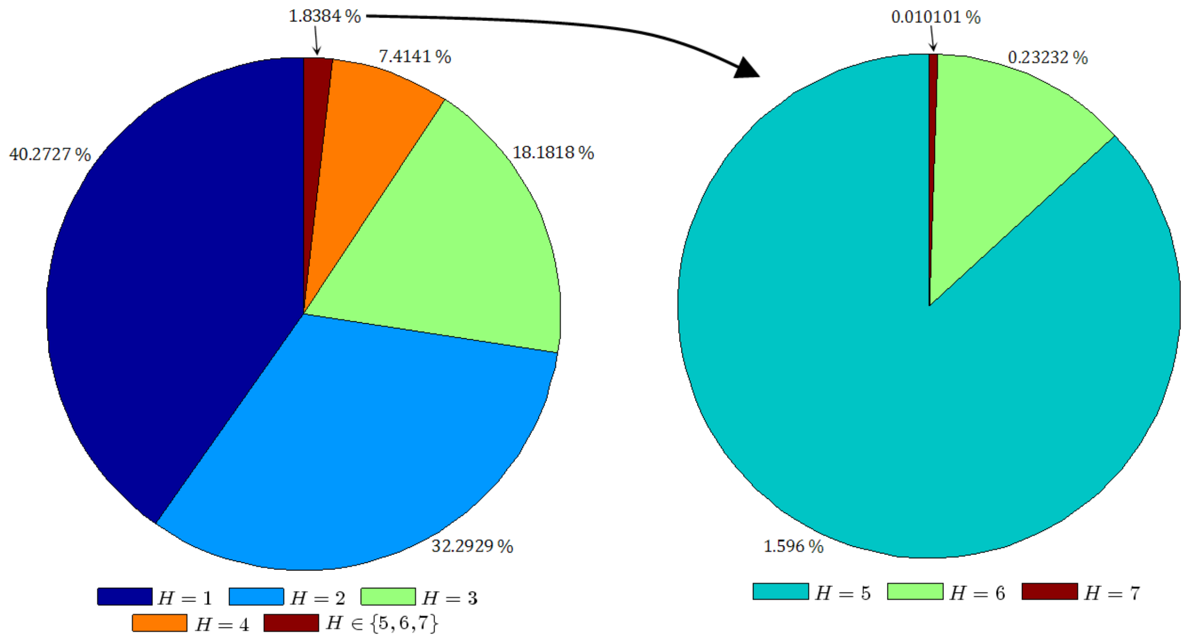

Figure 10 The pie chart of the number of efficient solutions

Source: Author

maximum edge length constraint. The problem with 12 disconnected trees has also occurred once by the same conditions, but it has six efficient solutions.

The single instance with $H = 7$ is a problem with 50 ships and the severest maximum edge length constraint ($\lambda = 1.25$), where, by the way, every disconnected subgraph has its efficient solution (Figure 11):

$$\widehat{N}_1^{**} = 2,\ \breve{l}_1^{**} = 4.6108,$$

$$\widehat{N}_2^{**} = 3,\ \breve{l}_2^{**} = 13.8964,$$

$$\widehat{N}_3^{**} = 4,\ \breve{l}_3^{**} = 30.2905,$$

$$\widehat{N}_4^{**} = 5,\ \breve{l}_4^{**} = 35.646,$$

$$\widehat{N}_5^{**} = 6,\ \breve{l}_5^{**} = 36.7046,$$

$$\widehat{N}_6^{**} = 7,\ \breve{l}_6^{**} = 39.4329,$$

$$\widehat{N}_7^{**} = 18,\ \breve{l}_7^{**} = 118.1354.$$

The best efficient solution has turned to connect seven ships (the ship numbering in Figure 11 is omitted for the sake of simplifying the visual perception) through the route whose length is $l^{**} = \breve{l}_6^{**} = 39.4329$, whereas the data transmission coverage connecting 18 ships (the largest subgraph on the right) would be the least efficient by $\breve{l}_7^{**} = 118.1354$. The other efficient solutions between those two are preferred in descending order:

$$\breve{T}_6^{**} \succ \breve{T}_5^{**} \succ \breve{T}_4^{**} \succ \breve{T}_3^{**} \succ \breve{T}_2^{**} \succ \breve{T}_1^{**} \succ \breve{T}_7^{**}. \quad (47)$$

The preference chain (47) corresponds to nearly equidistant distribution of the scores $\{v_h\}_{h=1}^{7}$ that vary between $v_6 = 0.4798$ and $v_7 = 0.6795$.

In addition to the number of disconnected trees (subgraphs) and the number of efficient solutions, another property of interest is the relationship between these numbers. This is about ratio Q/H which varies between 1 and 4 having 20 distinct values (Figure 12). As it is clearly seen in Figure 12, the likeliest case is when the number of efficient solutions is equal to the number of disconnected trees – it is 7988 instances out of 9900 (80.6869%). The remaining 19 values are distributed non-uniformly. The sorted distribution of their factual fractions by the number of occurrences is presented in Table 3 (the percentage values below 0.1 are shown with smaller font). Value $Q/H = 3/2$ being next to value $Q/H = 1$ has occurred 594 times out of 9900 (exactly 6%). The four succedent values of ratio Q/H sorted by the number of their occurrences are 4/3, 2, 5/4, 5/3, whose percentages drop from 4.4848% down to 1.6667%. The five succedent values 6/5, 7/4, 7/5, 5/2, 3 are of low probability due to their percentages are below 0.6% dropping from 0.596% down to 0.1515%. The remaining nine succedent values 7/3, 8/5, 7/6, 8/3, 9/5, 4, 9/4, 11/5, 7/2 having occurred 7 times and fewer are very unlikely. The single problem with seven disconnected subgraphs followed by two efficient solutions has occurred for 40 ships and $\lambda = 1.2$ (the constraint factor which is the closest to the severest maximum edge length constraint).

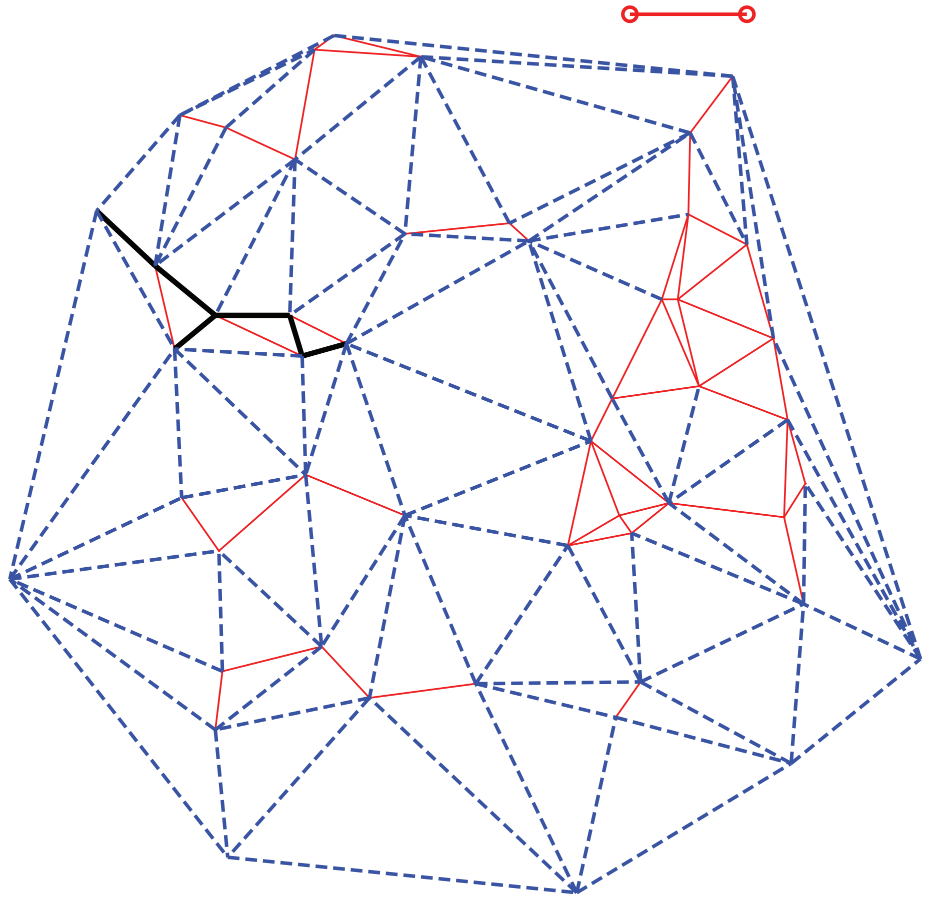

Figure 11 The problem with 50 ships and a maximum edge length constraint by λ = 1.25, where each of the seven disconnected subgraphs has its efficient solution (Q = 7 and H = 7); the severest maximum edge length constraint has removed 83 edges from the set of U = 138 edges (so P = 55), and the best efficient solution is the minimum spanning tree (highlighted bold) connecting seven ships

Source: Author

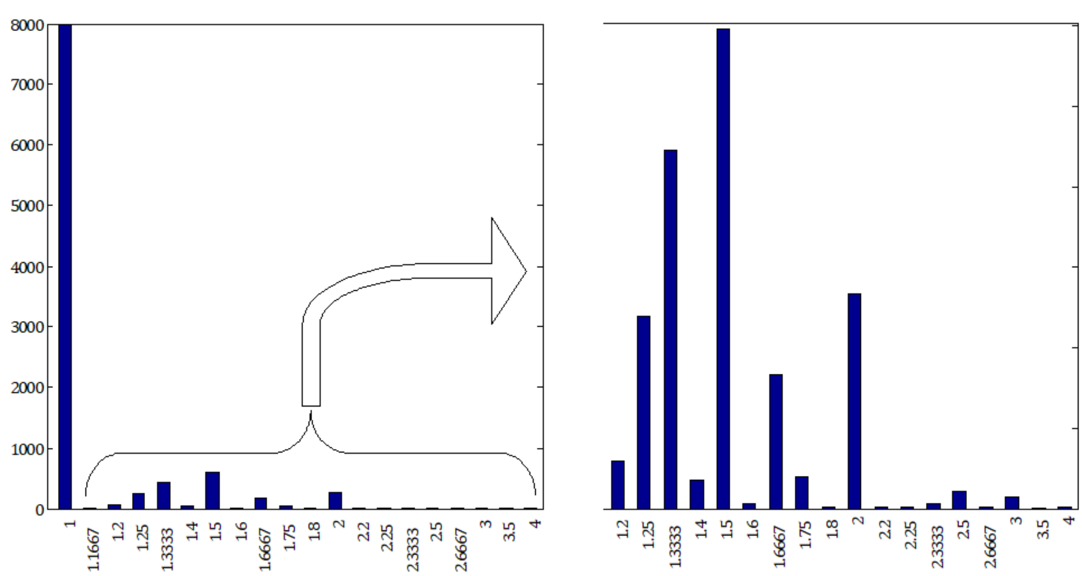

Figure 12 The distribution of occurrences of the values of ratio Q/H out of 9900 optimization problems

Source: Author

Table 3 The number of occurrences of 19 values of ratio Q/H

Ratio Q/H	**3/2**	**4/3**	**2**	**5/4**	**5/3**	**6/5**	**7/4**	**7/5**	**5/2**	**3**	**7/3**	**8/5**	**7/6**	**8/3**	**9/5**	**4**	**9/4**	**11/5**	**7/2**
Times occurred	594	444	265	238	165	59	40	36	22	15	7	7	7	3	3	2	2	2	1
Percentage	6	4.4848	2.6768	2.404	1.6667	0.596	0.404	0.3636	0.2222	0.1515	0.0707	0.0707	0.0707	0.0303	0.0303	0.0202	0.0202	0.0202	0.0101

Source: Author

Based on the statistics presented in Tables 1 – 3 and visualized in Figures 9, 10, 12, it is obvious that the suggested method has a strong practical impact. It is easily applicable for any number of ships. Factor α being an optional balancer for the two criteria and weighing the maximization of the number of connected ships is easily embedded also. The preference chain of the efficient solutions posted via trees $\left\{\breve{T}_h^{**}\right\}_{h=1}^{H}$ is another important implication from distances $\left\{v_h(\alpha)\right\}_{h=1}^{H}$. The matter is that if the best solution (the best minimum spanning tree) cannot be implemented by some reason, the next tree in the tree preference chain is selected (whose score is next to the least score). It is very significant for maritime data transmission coverage optimization because of versatility of maritime situations and conditions. For instance, the hotspot ship (say, the ship towards which a search-and-rescue operation must be accomplished) usually has the most powerful radio station. If it is out of order, some other ship must become the hotspot one. If there are two hotspot ships in Figure 7 example (say, ships 12 and 15), and the hotspot ship in solution (32) fails to broadcast and redirect for a while, then this solution is temporarily unrealizable, whereupon the broadcasting route by efficient solution (33) containing the other hotspot ship should be used instead.

7 Conclusion

Maritime data transmission coverage is volatile to a certain extent due to power and distance constraints, so maintaining optimal maritime data transmission coverage under power and distance constraints is quite an important task. The initial set of ship locations is triangulated, whereupon the edges exceeding the maximum edge length are removed. If the resulting graph has no disconnected nodes (ships), the solution is the minimum spanning tree. Otherwise, when there are two or more disconnected subgraphs, a minimum spanning tree is built for each of them, and the corresponding set of the efficient solutions is formed. Within this set, no minimum spanning tree exists that would either be shorter by connecting no fewer than a number of ships from an efficient solution or be not longer by connecting more than a number of ships from an efficient solution. The respective optimization problem, consisting in minimizing the broadcasting route length along with maximizing the number of ships communicating through the route, is solved by scalarizing the two criteria. The scalarization consists in standardizing the two criteria and calculating the distance of every achievable standardized efficient solution to the unachievable standardized solution.

The obtained statistical results of the computational simulation imply that the case of two or more disconnected subgraphs is about 60% probable (e. g., see Figure 9). Moreover, it is about 80% probable (see Figure 12) that the number of efficient solutions is equal to the number of disconnected subgraphs. These percentages indicate the factual usefulness of the suggested method to rationally minimize the broadcasting route length by simultaneously maximizing the number of connected ships. This is the main scientific and practical contribution to the field of maritime data transmission coverage, by which the minimum-energy broadcast routing is further rationalized.

The suggested optimization method has been studied for planar broadcast routing, though. For minimum-energy efficient communication among underwater research vehicles, a model of three-dimensional broadcast routing should be considered. The two-criterion optimization method might be extended in this way, to rationalize underwater data transmission coverage, where power and distance constraints are severer.

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