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Investigating the Effects of Cargo Weight and its Distribution on the Dynamic Performance of a High-Speed Planing Hull

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ABSTRACT

The aim of this article is to evaluate the effects of cargo relative weight and its distribution along the boat's length in addition to the relative water wavelength on the dynamic performance of a high-speed planing hull. Here, the dynamic performance is measured by the intensity of the boat's heave and pitch motions. The Zarnick's strip theory, which divides the vessel's hull into equal lateral sections, is used to study the applied forces on the vessel, and a MATLAB code is provided based on it. It is demonstrated that increasing the cargo weight and its distribution result in more heave and pitch, and the maximum amount of them are observed in the wave with a length of about 5 times the length of the boat. In addition, the interactive effects of cargo weight and its distribution on the heave and pitch motions are affected by the relative wavelength. Therefore, the more centralized cargo distribution is preferred to increase the dynamic performance of a heavy planing boat with cargo weights more than 50% of the boat's weight, while moving in short waves (λ/L<4); and for light cargo weight, less than 50% of the boat's weight, the favorite cargo distribution is broad. When the boat sails in long waves, the desired distribution is reversed.

1 Introduction

Nowadays, high speed vessels, regarding their speed, price, maneuverability, and vast applications, have acquired many usages and have been subjected under many researches. Meanwhile, improving the methods to predict sea-keeping and dynamic behaviour of planing hulls are indispensable. As a result, enhancing the stability and improving the performance of a planing hull in waves are important for high-speed boats.

These boats have complex dynamic behaviors at high speeds and suffer from high pitch and heave motions. Therefore, the design of a planing hull in the preliminary stages requires a correct prediction about behaviors of the boat, in order to prevent the occurrence of such undesirable motions. The changes in the distribution and the amount of cargo weight on the boat can affect the vessel behavior. Meanwhile, numerical methods are highly beneficial to analyze ships motions and their effective parameters.

Motions of the high-speed planing hulls have been analyzed in various researches in three fields: laboratory methods, numerical methods and analytical models. Yosefi et al. comprehensively reviewed the hydrodynamic analytical methods for high-speed planing hulls since 2013 [1].

Some researchers have focused on planing boats with stepped hulls to provide more comfortable cruise with higher speed. Trimulyono et al. used computational fluid dynamics (CFD) to modify a two steps hull with variations in the position of the steps. Their research aimed to determine the effect of the first and second step positions on the total resistance, dynamic trim, and dynamic sinkage [2]. The other research, by Avci and Barlas, aimed to evaluate the optimal longitudinal position of a single transverse step. They experimentaly studied the resistance properties of the hull with four different configurations and determined the optimum longitudinal location of the transverse step [3].

One of the other methods to reduce planing craft resistance is to use the interceptor. Samuel et al. used CFD for calm water conditions to analyze the hydrodynamics of a planing hull with an interceptor. They showed that the interceptor is remarkably useful in trim control and planing hull drag reduction [4].

In analytical models, Sahin et al. presented a mathemathical 2D model, representing the vertical motion of the prismatic planing vessel. They presented an automatically controllable system to control the interceptor in order to minimize the total resistance and enhance comfort for high-speed planing vessels [5].

Many studies on planing hulls have been developed based on the impact of a wedge on the surface of the water. Von Karman reduced the level of the problem from three dimensions to two dimensions and simplified the vessel cross sections into wedges [6]. According to Von Karman momentum theory, when an object enters water, its momentum is divided between the body and the fluid around it, and the forces acting on the object can be measured by the rate of momentum change. Instead of considering a wedge, Wagner reduced the problem to a sheet entering the water surface and varied the width of the sheet with time [7]. Clement and Blount performed a series of tests on the TM-62 vessels. In these experiments the variable visual aspect ratio (length-beam ratio) was evaluated on a base model with prismatic end and a deadrise angle of 12.5 degrees [8]. They found that the amount of trim and heave depend on the geometry of the body, the forward speed, and the longitudinal location of the center of gravity. The experiments on a set of prismatic body boats with a fixed deadrise angle along the body in calm water and regular waves were performed by Fridsma [9]. Fridsma also studied the behavior of a planing hull facing irregular waves [10]. Katayama et al., considering linear and nonlinear motions, performed some model experiments with Froude numbers of 2 to 5 in calm water and in regular waves [11]. They also calculated hydrodynamic coefficients from laboratory and theory (based on the potential theory), and compared the results of the experiments with the results of the dynamic simulations. These authors concluded that the nonlinear strip theory method with a proper calculation for hydrodynamic forces can predict the vertical motions of a planing boat with sufficient accuracy for scientific purposes.

Garme performed experiments based on the Fridsma's model in calm waters and regular waves with different velocities [12]. His simulation results showed a good correlation between heave, velocity, and vertical acceleration values, but the results for pitch motions were much higher than actual values. Moreover, Ghadimi et al. presented a mathematical model to investigate the effect of variable deadrise angle and trim angle of a planing hull on its performance [13].

Furthermore, to calculate hydrodynamic forces on a planing hull, Wang et al. presented a new computational method based on computational fluid dynamics [14]. After that, Martin investigated the instability of a planing hull which is caused by heave and pitch motions in calm water [15]. He developed a method for predicting the surge, heave, and pitch motions by which a planing hull encounters porposing instability. Based on Martin's work, Zarnick developed a nonlinear mathematical model using strip theory [16]. This mathematical model was developed for a prismatic planing hull with a fixed deadrise angle at high speeds and in regular waves. Zarnick used the strip theory to determine the coefficients in the motion equations using a combination of theoretical and experimental relationships. He also assumed that the water wavelength can be larger than the vessel's length and the slope of the wave is small. The Zarnick's 2D strip theory was used by Sayeed et al. to predict the vertical motions of a planing hull in head waves [17]. Their results were capable to be implemented in the simulator for training purposes. Next, Keoning considered trim and heave of a high-velocity boat and developed Zarnick's method using special relations [18]. He also studied the distribution of hydrodynamic lifting forces along the body with nonlinear added-mass and wave-imposed force in regular and irregular waves. He proposed a computational model incorporating the main nonlinear parameters to predict the wave effects with high accuracy. Besides, Van Dayzen developed Keuning model to three degrees of freedom for surge, heave, and pitch motions in regular and irregular waves [19]. In addition, Chiu and Fujino added an elastic parameter to the Zarnick theory by considering the effects of inertia [20]. They omitted the quadratic and higher order terms in the equations of motion and evaluated the hydrodynamic coefficients for oscillating motions. The completely nonlinear equations proposed by Zarnick were transformed to Taylor series by Hicks et al. [21]. They also identified areas of dynamic response and the effect of second-order terms on the boat progress. Moreover, Blake proposed a linear model in a frequency range based on Martin's studies and found that it would be necessary to consider the time-dependent wetting length to improve predicting the motion behavior of a boat [22]. Thereupon, Lewis et al. developed Blake's numerical model, which itself was based on the Zarnick's nonlinear strip theory [23]. They found that their numerical model can predict motions of the larger ships.

Niazmand Bilandi et al. developed a 2D+T theory to calculate the boat's dynamics and heave and pitch motions for a double-stepped planing hulls [24]. Their results showed the appropriate accuracy of the method

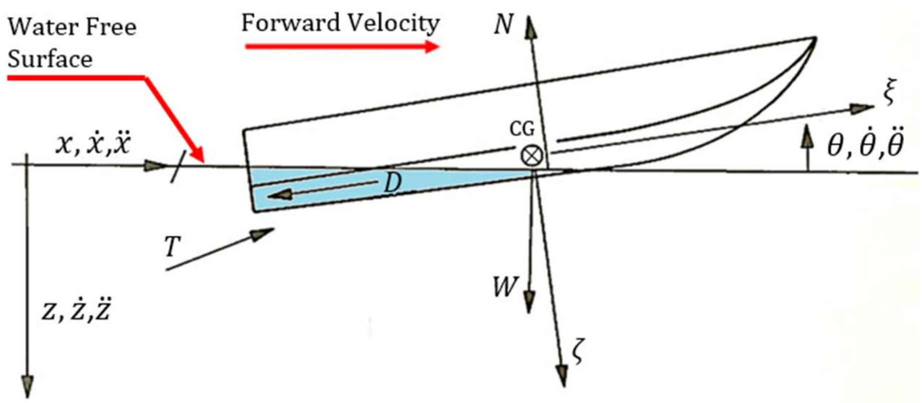


Figure 1 Reference and moving coordinate frames for a planing boat

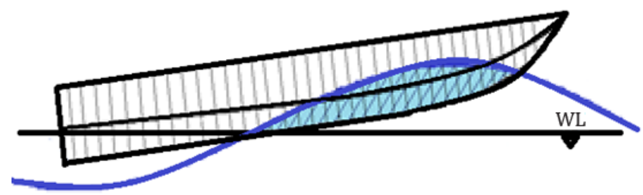
Source: Authors

and its readiness to predict the behavior of a double-stepped planing hull in rough water. Ghadimi et al. presented an analytical method to simulate the roll motion of a flying boat and compared it with experimental data [25]. However, Tavakoli et al. [26, 27], using mathematical simulation, studied the motion of a planing hull in the direction of heave, pitch, and roll, and showed that the range of the motions have no significant effect on the boat's hydrodynamic. Also, the results of this simulation showed that the range of these motions have no significant effect on hydrodynamic coefficients. According to the Hoseinzadeh et al. [28], reducing the coefficient of added mass led to a faster vehicle and decreased the heave and pitch, which promote the accordance between numerical outcomes and Fridsma's experimental results.

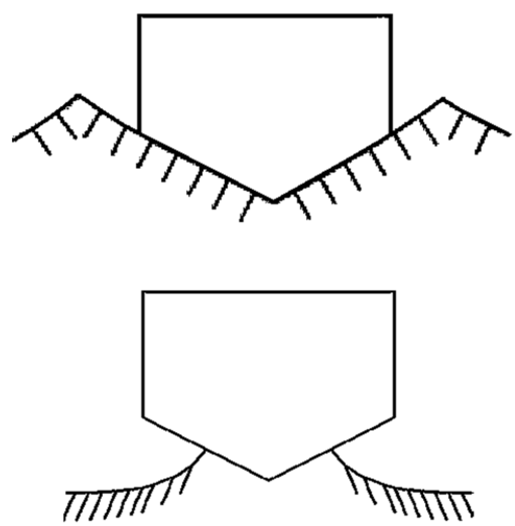
In this research, Zarnick's two-dimensional method along with the improvement by Hoseinzadeh is used and a code is developed. After ensuring the accuracy of the software in comparison with Fridsma experimental data, the effects of the cargo weight and its distribution on the behavior of a planing hull are investigated.

2 Mathematical Model

The model assumptions are alike with Zarnick's model [16], which formulated a nonlinear mathematical model for planing boats with fixed deadrise angle in regular waves and a constant forward velocity. According to Fig. 1, the model considers the reference xyz coordinate system and a moving ξχζ coordinate system that its origin is placed at the center of gravity of the boat. The hull is divided into cross-sectional strips to provide separately calculation of hydrodynamic forces over each section. Fig. 2a shows the intersection of the boat hull sections and water surface. Fig. 2b also shows interaction of the body wedge strips with water, which can arise two conditions.



(a)



(b)

Figure 2 (a) The longitudinal and (b) transverse interactions of the vessel with water.

Source: [16]

The forces acting on each section depend on the depth of penetration of each section in water. The total hydrodynamic force on the boat's body is obtained by integrating the forces on each section. As a result, the forces are calculated and their moments are determined.

2.2 Equations of motion

The dynamic equations of motion for a planing hull, as seen in Fig. 1, include:

$$M\ddot{x}_{CG} = T_x + F_x - D\cos\theta$$
$$M\ddot{z}_{CG} = T_z + F_z + D\sin\theta + W \quad (1)$$
$$I\ddot{\theta} = T_x x_P + F_\theta - Dx_d$$

The normal velocity (V) and parallel velocity (U) regarding to the keel, are:

$$U = \dot{x}_{CG}\cos\theta - (\dot{z}_{CG} - w_z)\sin\theta$$
$$V = \dot{x}_{CG}\sin\theta - \theta\dot{\xi} + (\dot{z}_{CG} - w_z)\cos\theta \quad (2)$$

The submergence of each section of the boat's hull is (h):

$$h = z_{CG} - \xi\sin\theta + \zeta\cos\theta - r \quad (3)$$

In which, r is the wave elevation for regular head waves according to Equation (4):

$$r = r_0\cos[k(x + ct)] \quad (4)$$

where r_0 represents the wave amplitude, k is the wave number, and c is the wave celerity. The hydrodynamic forces acting on the body are obtained by integrating the forces computed for each section. The governing equations that determine the motion of the planning hull can be written as Equation (5) [16]:

$$\begin{bmatrix} M + M_a sin^2\theta & M_a\sin\theta\cos\theta & -Q_a\sin\theta \\ M_a\sin\theta\cos\theta & M + M_a cos^2\theta & -Q_a\cos\theta \\ -Q_a\sin\theta & -Q_a\cos\theta & I + I_a \end{bmatrix}\begin{bmatrix}\ddot{x}_{CG}\\ \ddot{z}_{CG}\\ \ddot{\theta}\end{bmatrix} = \quad (5)$$
$$= \begin{bmatrix} T_x + F'_x - D\cos\theta \\ T_z + F'_z + D\sin\theta + W \\ Tx_P + F'_\theta - Dx_d \end{bmatrix}$$

2.3 Hydrodynamic force and moment

The corresponding equation for F'_x, F'_z and F'_θ can be written as equations (6-8):

$$F'_x = \Big\{M_a\dot{\theta}(\dot{z}_{CG}\sin\theta - \dot{x}_{CG}\cos\theta) +$$
$$+ \int_l m_a\frac{dw_z}{dt}\cos\theta d\xi - \int_l m_a w_z\dot{\theta}\sin\theta d\xi -$$
$$- \int_l m_a V\frac{\partial w_z}{\partial\xi}\sin\theta d\xi + \int_l m_a U\frac{\partial w_z}{\partial\xi}\cos\theta d\xi - \quad (6)$$
$$- UVm_{a_{stern}} - \int_l V\dot{m}_a d\xi - \rho\int_l C_{D,c}bV^2 d\xi\Big\} sin\,\theta$$

$$F'_Z = \Big\{M_a\dot{\theta}(\dot{z}_{CG}\sin\theta - \dot{x}_{CG}\cos\theta) +$$
$$+ \int_l m_a\frac{dw_z}{dt}\cos\theta d\xi - \int_l m_a w_z\dot{\theta}\sin\theta d\xi -$$
$$- \int_l m_a V\frac{\partial w_z}{\partial\xi}\sin\theta d\xi + \int_l m_a U\frac{\partial w_z}{\partial\xi}\cos\theta d\xi - \quad (7)$$
$$- UVm_{a_{stern}} - \int_l V\dot{m}_a d\xi - \rho\int_l C_{D,c}bV^2 d\xi\Big\}\cos\theta -$$
$$- \int_l a_{BF}\rho gbAd\xi$$

$$F'_\theta = -Q_a\dot{\theta}(\dot{z}_{CG}\sin\theta - \dot{x}_{CG}\cos\theta) -$$
$$- \int_l m_a\frac{dw_z}{dt}\cos\theta\xi d\xi + \int_l m_a w_z\dot{\theta}\sin\theta\xi d\xi -$$
$$- \int_l m_a V\frac{\partial w_z}{\partial\xi}\sin\theta\xi d\xi - \int_l m_a U\frac{\partial w_z}{\partial\xi}\cos\theta\xi d\xi + \quad (8)$$
$$+ m_a UV\xi_{stern} + \int_l m_a UVd\xi + \int_l V\dot{m}_a\xi d\xi -$$
$$- \rho\int_l C_{D,c}bV^2 d\xi + \int_l a_{BM}\rho gbAcos\,\theta\xi d\xi$$

In these relationships, A is the cross-sectional area of each strip, a_{BF} is a correction factor equal to ½ [16], and a_{BM} = ½a_{BF}. $C_{D,C}$ represents the cross-flow drag coefficient equal to 1.33 cosβ. Note that the integration terms are considered only over the wetted length of the hull (l).

The amount of added-mass of a wedge-shaped section is determined by Equation (10):

$$m_a = k_a\frac{\pi}{2}\rho b^2 \quad (9)$$

Where, k_a is the two-dimensional added-mass coefficient and ρ is the density of the water. The parameter b is the half beam which is calculated according to: $b = d$ cotβ, but when the water pile-up is considered, it is calculated as follow:

$$b = C_{pu}d\,cot\beta \quad (10)$$

Here, the coefficient of C_{pu} is calculated by:

$$C_{pu} = \frac{\pi}{2} - \beta\left(1 - \frac{2}{\pi}\right) \quad (11)$$

Zarnick [16] suggested k_a = 1; however, to establish more comparable numerical data with experimental, Hoseinzadeh [28] offered a relationship based on the deadrise angle for the added-mass coefficient (Equation (12)).

$$k_a = 0.32 + \left[\frac{0.73}{cos\beta}\times\left(1 - \frac{2\beta}{\pi}\right)^{9/2}\right] \quad (12)$$

The M_a, Q_a and I_a are as equation (14):

$$M_a = \int_l m_a d\xi$$
$$Q_a = \int_l m_a \xi d\xi \quad (13)$$
$$I_a = \int_l m_a \xi^2 d\xi$$

It is assumed that the high speed planing boat has a constant velocity, and the resultant of drag force (D) and thrust (T) are small in comparison with hydrodynamic forces. As a result, the equations of motion can be written as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & M + M_a cos^2\theta & -Q_a \cos\theta \\ 0 & -Q_a \cos\theta & I + I_a \end{bmatrix} \begin{bmatrix} \ddot{x}_{CG} \\ \ddot{z}_{CG} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ F'_z + W \\ F'_\theta \end{bmatrix} \quad (14)$$

3 Solving Equations of Motion

In this section, the equations of motion for a planing boat are solved using a computer code and in order to be validated, the results of the simulation are compared with the results of the Fridsma's boat model [9]. Then, the dynamic behaviors of the vessel are studied through changing the distribution and the amount of cargo weight.

The obtained equations of motion, which are combination of two second-order nonlinear differential equations, must be solved over time using standard numerical techniques. Here, a numerical technique is used which is a predictive-corrective method and suggested by of Adams-Bashfort-Multon. Predictive-corrective methods refer to a set of tricks for solving differential equations that use two different predictive and corrective formulas. A predictor is an explicit formula used to determine an initial estimate of the answer y_{i+1}. Since the predictor is an explicit formula, the value of y_{i+1} is calculated from the known answer at the previous point $(x_i \cdot y_i)$ (single-step method) or several previous points (multi-step method). After finding the initial estimation of y_{i+1}, the corrector is used for calculating a new and more accurate value for the answer y_{i+1}.

3.1 Predictive-Corrective Method

In the third-order predictive equation, the initial estimate for y_{i+1} would be:

$$y_{i+1}^{(1)} = y_i + \frac{h}{12}(23f(x_i.y_i) - 16\,f(x_{i-1}.y_{i-1}) + \\ + 5\,f(x_{i-2}.y_{i-2}) \quad (15)$$

and the corrective equation is:

$$y_{i+1}^{(k)} = y_i + \frac{h}{12}\left[5f_{i+1}^{(k-1)} + 8\,f_i - f_{i-1}\right], \quad k = 2.3.\ldots \quad (16)$$

The terms in Equation (16) are explained as follow:

$$f_i = f(x_i.y_i)$$
$$f_{i-1} = f(x_{i-1}.y_{i-1}) \quad (17)$$
$$f_{i+1}^{(k-1)} = f\left(x_{i+1}.y_{i+1}^{(k-1)}\right)$$

Therefore, to perform the analysis, a code for extracting the values of heave and pitch motions for the center of gravity of a planing hull is written in MATLAB software. In this code, the equations of motion are solved over time using the Adams-Bashfort-Moulton numerical technique. The procedute for calculating the heave and pitch motions of the boat is indicated in Fig. 3.

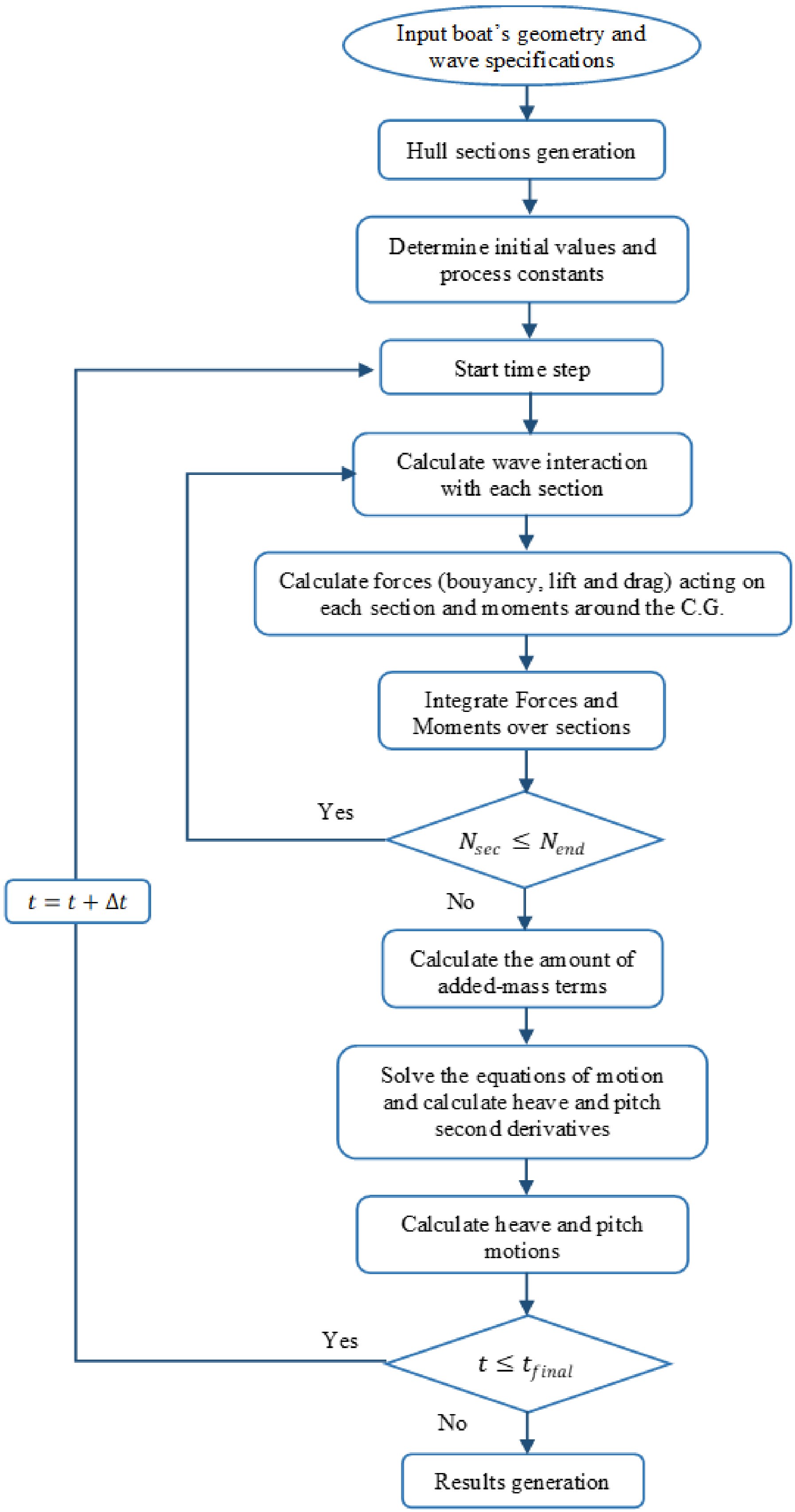


Figure 3 Flowchart of mathematical calculation

Source: Authors

4 Results and Validation

In this section, the accuracy of the code is shown through comparing the results of this numerical analysis with the heave and pitch motions resulted from Fridsma's experiments. As shown in Fig. 4, the idealized prismatic hulls with different deadrise angles, designed by Fridsma, move in a variety of regular waves.

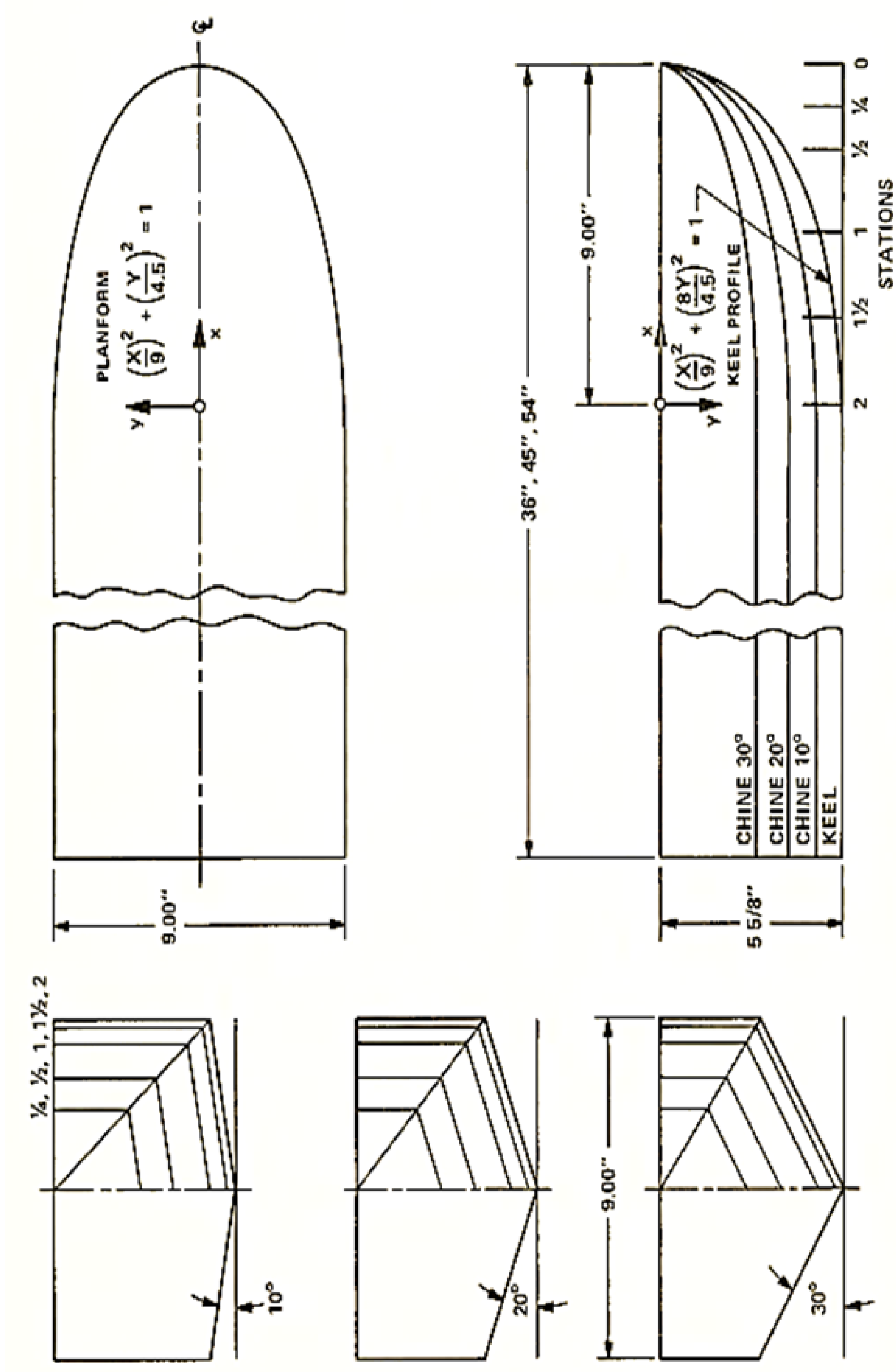


Figure 4 Fridsma's model for a planing boat

Source: [16]

4.1 Specifications of the planing hull studied by Fridsma

Principal dimensions of the planing boat studied by Fridsma are given in Table 1.

Table 1 Principal dimensions of the planing boat

L	3.75 ft (1.143 m)
B	0.75 ft (0.229 m)
W	16.42 lb (7.45 kg)
V	19.6 ft/s (5.97 m/s)
β	10º, 20º and 30º
r_g/L (pitch radius of gyration relative to length)	0.255
LCG/L (longitudinal center of gravity relative to length)	62% (from bow)

Source: [16]

Furthermore, considering different wave lengths and a constant wave height of 1 inch (2.54 cm), the beam Froude number was CV = 4 and H/B = 0.111. The wave lengths regarded in the Fridsma's experiments are given in Table 2.

Table 2 Wave lengths of Fridsma's tests

No.	λ/L	C_λ
1	6	0.049
2	4	0.073
3	3	0.097
4	2	0.146
5	1.5	0.195
6	1	0.292

Source: [16]

4.2 Code validation

Fig. 5 to Fig. 8 show the results of response amplitude operator (RAO) for heave and pitch motions of the planing hull with deadrise angles of 20º and 30º versus $C_\lambda = \frac{L}{\lambda}[C_\Delta/(L/B^2)]^{1/3}$. As can be seen, the developed code is able to generally predict the behaviors of the planing boat and it is confirmed by the experimental data.

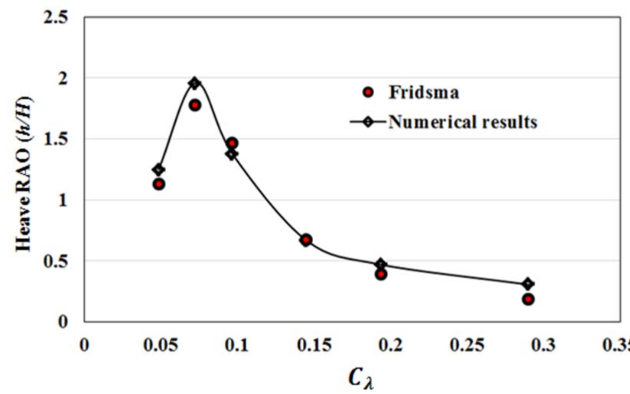


Figure 5 Heave RAO of the planing hull with C_V = 4, β = 20° and H/B = 0.111.

Source: Authors

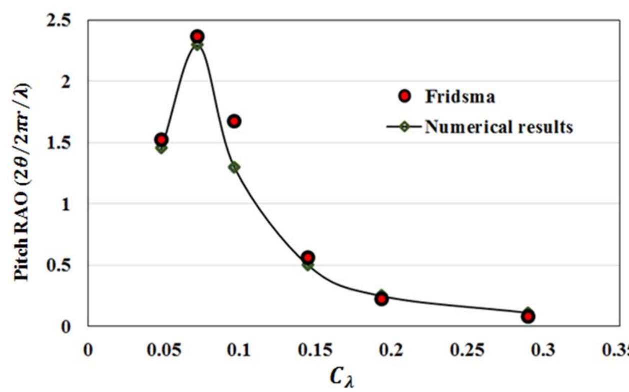


Figure 6 Pitch RAO of the planing hull with C_V = 4, β = 20° and H/B = 0.111.

Source: Authors

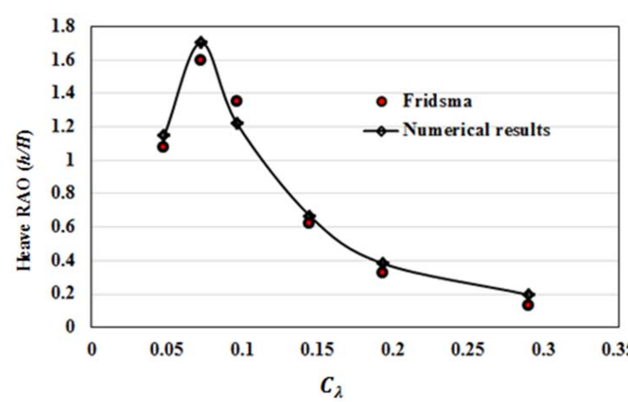


Figure 7 Heave RAO of the planing hull with C_V = 4, β = 30° and H/B = 0.111.

Source: Authors

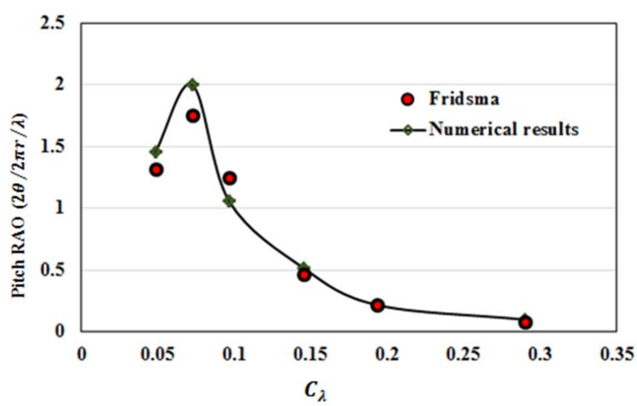


Figure 8 Pitch RAO of the planing hull with C_V = 4, β = 30° and H/B = 0.111.

Source: Authors

5 The Effects of Cargo Weight and its Distribution

What is being considered here is the effect of amount and distribution of cargo loads on the boat's heave and pitch motions. For this purpose, the added cargo weight on the boat in each distribution is increased by 10, 20, 30, 40, 50, 75 and 100% of the weight of the vessel. Besides, the cargo loads are symmetrically distributed around C.G. in a manner that the center of gravity does not shift along the boat's length. Fig. 9 shows the distribution of cargo loads are along the boat.

The pitch radius of gyration for the added loads is $rw_g = \alpha r_g$. As shown in Fig. 9, in order to vary the cargo load distribution, the radius of gyration is changed by changing α coefficient for each analysis. The amount of α is selected as 0.25, 0.5, 0.75, 1, and 1.5.

In Fig. 10 the results of heave RAO and pitch RAO for deadrise angle of 20° are plotted respect to λ/L. Here, different wavelengths from 1 to 6 times of the boat's length are evaluated and the added cargo loads are changed from 10% to 100% of the planing boat's weight.

It can be seen in Fig. 10 that, in general, the changes of heave and pitch motions are similar to each other respect to the relative wavelength. So that by increasing the value of λ/L, the oscillations of the heave and pitch motions increase and form a maximum at λ/L around 5, and after that, the amplitude of the vessel's oscillations decreases. In this regard, since with increase of λ/L more than 5, the motions of the boat become more concordant with the wave surface, the boat is expected to experience less fluctuations compared to the water surface.

According to results as shown in Fig. 11, it seems that λ/L greater than 7 does not have much effect on heave and pitch motions for the planing boat. In those waves with large relative wavelengths, the boat's heave and pitch motions are relatively constant and are not affected by factors such as cargo weight and its distribution. Therefore, expecting a maximum in the heave and pitch motions can be reasonable around the λ/L=5. Another reason for the occurrence of this maximum around the λ/L=5 can be the amount of available time that a wave applies upward force to the boat or separates from boat's bottom so that the boat is pulled down with its weight. In waves with λ/L less than 4, before reaching the maximum of the heave and pitch, the wave applies the opposing forces against ongoing heave and pitch motions. These available times for heave and pitch motions are longer in waves with λ/L of about 5. Therefore, in waves with λ/L=5, the heave and pitch motions reach their maximum.

Besides, local maximums for heave and pitch motions are observed at λ/L=2. The reason for this is related to the collision mode between the profile of a sine wave and the boat's bottom.

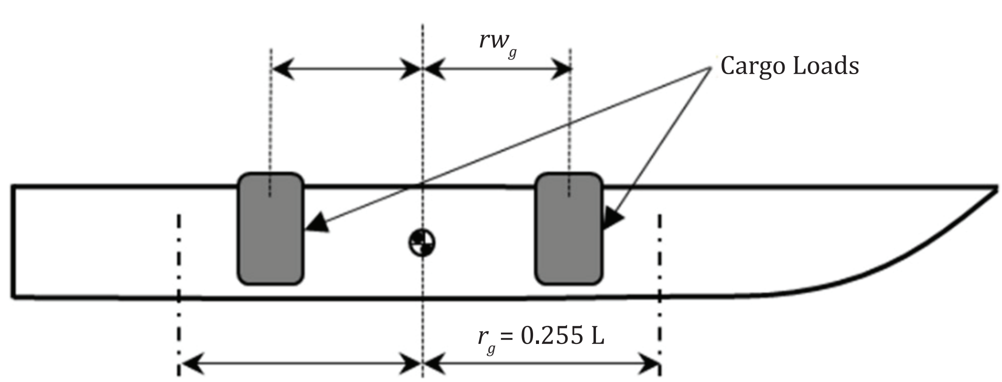


Figure 9 The pitch radius of gyration of the added weight

Source: Authors

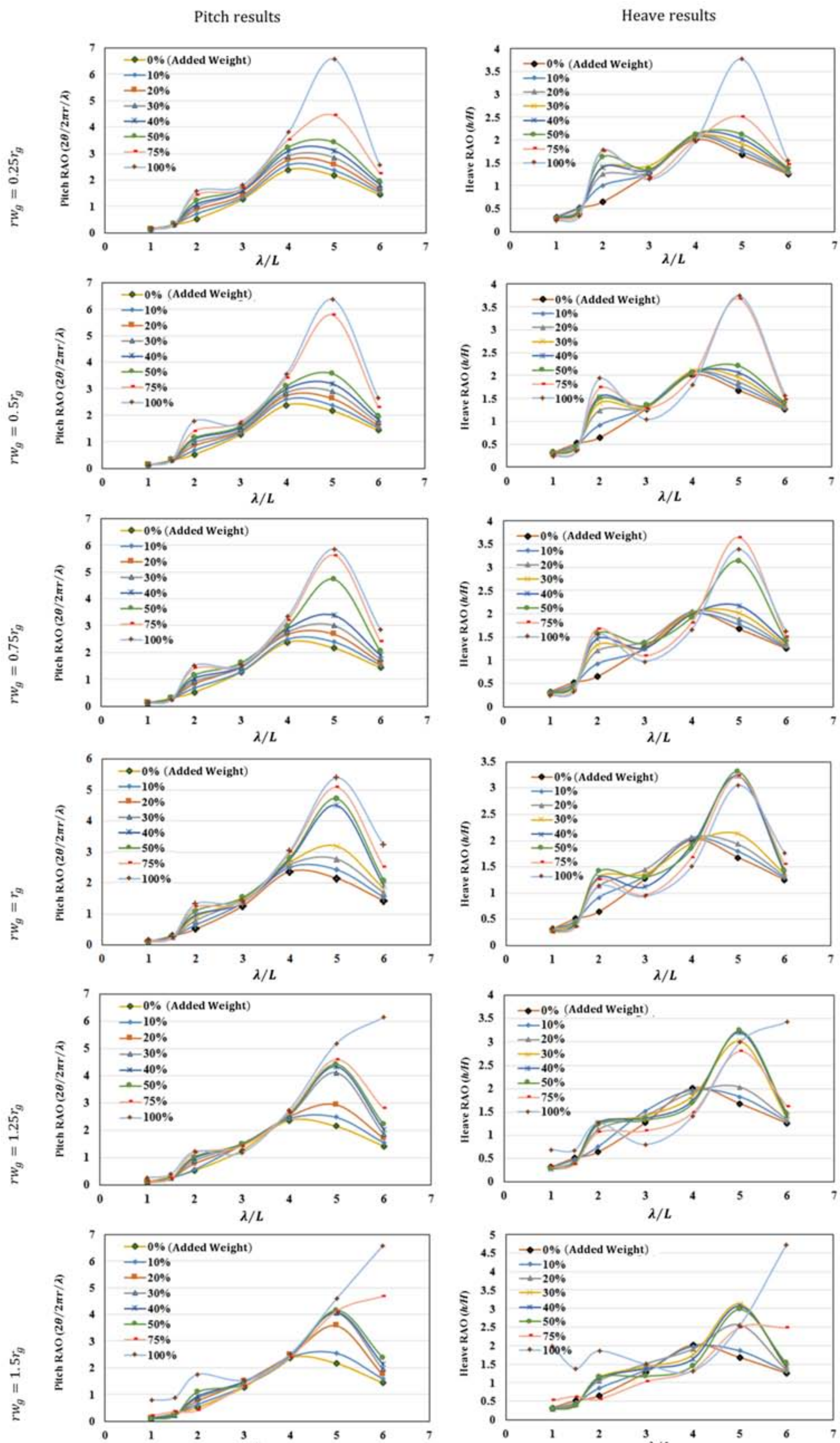


Figure 10 Heave RAO and pitch RAO for the boat with the deadrise angle of 20° against waves with different relative wavelengths and under different added cargo weight with various distributions.

Source: Authors

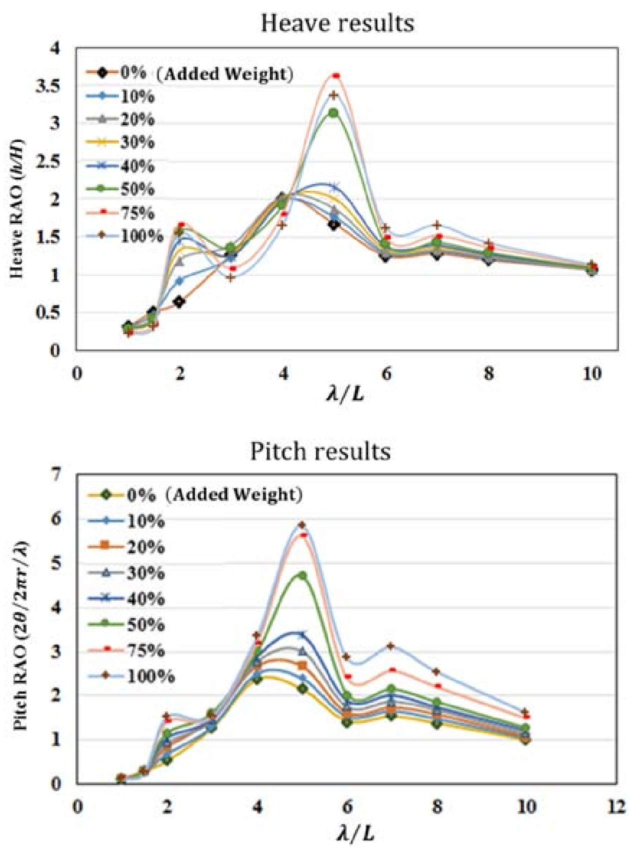


Figure 11 The heave RAO and pitch RAO of the planing boat against waves with longer relative wavelengths and different added weights at $rw_g = 0.75\ r_g$

Source: Authors

It can also be seen in Fig. 10 that the ranges of heave and pitch fluctuations are affected by the distribution of cargo weights. So that with increasing value of rw_g, often, the amplitude of heave and pitch motions increases. Increasing the rw_g also causes the maximum heave and pitch fluctuations to occur in waves with higher λ/L. For example, at $rw_g = 0.5\ r_g$, the maximum value of heave and pitch are observed in waves with λ/L less than 5; however, by increasing the rw_g to 1.25 r_g, the maximum oscillations of heave and pitch are observed in waves with λ/L more than 5. The reason of this can also be related to the increase in the angular momentum of the boat due to the wider distribution of the cargo loads along the boat. As the angular momentum increases, the boat's resistance against twisting motions increases, and therefore, the stronger waves with λ/L more than 5 can lead more pitch motion. In addition, the amount of heave is also affected by the angular momentum of the boat. Considering more resistance to pitch motions for the boat with more rw_g, when a wave crest with λ/L greater than 5 strikes the bow or stern of the boat, it vertically moves the boat, and as a result, the amount of heave increases.

The amplitudes of the heave and pitch are also affected by the cargo loads. So that the increase in the amount of relative additional load, more than 75% comparing to less than 50%, causes a growth about 100% in the maximum amplitude of the heave and pitch motions. Furthermore, it can be seen in Fig. 10 that as a result of increasing the amount of added loads, the maximum oscillations of heave and pitch are transferred to waves with λ/L greater than 5. On the other hand, if there are few cargoes loaded in the boat (<20%), the maximum value of heave and pitch shift to λ/L=4.

Besides, the amount of additional weight and its distribution have mutual effects that more intensify heave and pitch motions. It is obvious that for λ/L=4, the intensive fluctuations and unsetting condition, i.e. pitch RAO>4 and heave RAO>2.5, are observed at rw_g = 0.25 r_g and when the cargo relative weight is more than 75%. However, when rw_g increases, this situation is seen for less added weights. For example, for λ/L=4, when rw_g = 1.5 r_g, the instable condition is established for the added relative weights even more than 30%. Moreover, for λ/L=5 and medium and low added weights, vast load distribution clearly increases the range of boat oscillations. Accordingly, increasing the amount of both added weight and rw_g increase the amount of heave and pitch.

Despite the increasing effect of rw_g on the pitch values in various relative wavelengths, according to Fig.10, increased rw_g causes less pitch motion for heavier boats. In addition, the more cargo weight is loaded, the more pitch motion is observed for more centralized cargo. However, when the planing boat is loaded more than 100% of its weight, its heave and pitch motions become intense and instable in long wavelengths (λ/L>4).

Moreover, it seems that λ/L=4 is a converging spot for the heave motions. According to Fig. 10, all values of heave RAO reach around 2 at λ/L>4. Nevertheless, increasing cargo weight and its distribution result in some deviation in the amount of heave RAO to the values less than 2.

A question may arise here is what could be the best cargo distribution (rw_g) for a boat to have minimal heave and pitch motions. In general, it can be said that if a boat sail with relative cargo loads less than 50% of the boat's weight, for small waves with λ/L<4, it is better to spread the cargo widely along the boat (larger rw_g), and for long waves (λ/L>5), it is better to arrange the cargo with a concentrated distribution (smaller rw_g).

In another case, if the cargo is loaded more than 50% of the boat's weight, for λ/L<4, it is better to arrange the cargo near the mass center of the boat (smaller rw_g), and if the vessel moves in long waves (λ/L>5), in order to minimize the heave and pitch motions, it is beneficial to spread the loads along the boat's length (larger rw_g).

6 Conclusion

In this study, using the Zarnick's strip theory and the geometry used in Fridsma's research, the dynamic analysis of a planing hull in regular waves was performed. The effects of cargo weight and its distribution on the values of the boat's heave and pitch motions were evaluated against different relative wavelengths. The results obtained from this study can be expressed as follow:

When the relative wavelength is about 5 times the boat's length, the forces interactions can lead to severe heave and pitch motions. By increasing the value of λ/L, the heave and pitch motions increase and form a maximum at λ/L around 5 and then decrease. Besides, a local maximum for heave and pitch motions are observed at λ/L=2.

The ranges of heave and pitch fluctuations are affected by distribution of cargo weight. So that with increasing rw_g, often, the amounts of heave and pitch motions raise. Increasing rw_g also causes the maximum heave and pitch fluctuations to occur in waves with higher λ/L.

The increase in the cargo load, more than 75% of the boat's weight comparing to less than 50%, causes a growth about 100% in the maximum amplitude of the heave and pitch motions.

If there are few cargo loaded in the boat (<20%), the value of heave and pitch motions are less affected by added weight distribution, and their maximums are formed near λ/L=4.

The distribution of the cargo weight (rw_g) shows the increasing effect on the pitch values in various wavelengths. However, increased rw_g causes less pitch motion for heavier boats. In addition, the more cargo weight is loaded, the more pitch motion is observed for more centralized cargo. However, when a boat is loaded more than its weight, its heave and pitch motions become intense and instable in long wavelengths.

In order to minimize the heave and pitch motions, when the boat is sailing with light cargo in short relative wavelingths (λ/L<4), it is better to spread the cargo widely along the boat (larger rw_g), and in long waves (λ/L>5), it is better to arrange the cargo with a concentrated distribution (smaller rw_g). In another case, when a boat with heavy relative cargo loads (>50%) moves in short waves (λ/L<4), the less rw_g is more preferred, and in long waves (λ/L>5), it is beneficial to spread the loads along the planing boat's length (larger rw_g).

Nomenclature

a_{BF}	Buoyancy correction factor
a_{BM}	Moment correction factor
b	Half beam
B	Beam of the hull
c	Wave speed
C_V	Beam Froude number, $\frac{U}{\sqrt{gB}}$
$C_{D,C}$	Cross flow drag coefficient
C_{pu}	Pile-up coefficient
C_Δ	Load coefficient, $\frac{W}{\rho g B^3}$
C_λ	Wave length coefficient, $\frac{L}{\lambda}[C_\Delta/(L/B^2)]^{1/3}$
d	Depth of penetration of each section
D	Friction drag
$\vec{F}$	Force vector
F_x	Hydrodynamic force in x direction
F_y	Hydrodynamic force in y direction
F_θ	Hydrodynamic moment about pitch axis
f_b	Buoyancy force of each section (Sectional buoyancy force)
f_{CD}	Hydrodynamic force of each section (Sectional viscous lift associated with the cross flow drag of a calm water penetrating wedge)
g	Acceleration due to gravity
H_w	Wave height
I	Pitch moment of inertia
I_a	Added pitch moment of inertia
k	Wave number
k_a	Added mass coefficient
l	Wetted length of the hull
L	Vessel's length
LCG	Longitudinal center of gravity
m_a	Added mass associated with each section
M	Vessel's mass
M_a	Vessel's added mass
N	Vertical hydrodynamic force
r	Wave profile
r_0	Wave amplitude
r_g	Hull radius of gyration
rw_g	Added weight radius of gyration
T_x	Thrust component in x direction
T_z	Thrust component in z direction
U	Boat velocity parallel to keel
V	Boat velocity perpendicular to keel
VCG	vertical center of gravity
w	Weight of each section of the vessel
W	Weight of the boat
w_z	Wave orbital velocity in vertical direction
$\vec{\dot{x}}$	State variable vector
x_c	Distance from CG to center of pressure for normal force
x_d	Distance from CG to center of action for drag force
x_p	Moment arm of thrust about CG
$x_{CG}\cdot\dot{x}_{CG}\cdot\ddot{x}_{CG}$	Surge displacement, velocity and acceleration
$z_{CG}\cdot\dot{z}_{CG}\cdot\ddot{z}_{CG}$	Heave displacement, velocity and acceleration
β	Deadrise angle
$\theta\cdot\dot{\theta}\cdot\ddot{\theta}$	Pitch angle, velocity and acceleration
$x\cdot y\cdot z$	Reference coordinate system
$\xi\cdot\chi\cdot\zeta$	Body coordinate system
λ	Wavelength
ρ	Density of water
ν	Wave slope

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References

[1] R. Yousefi, R. Shafaghat, and M. Shakeri, "Hydrodynamic analysis techniques for high-speed planing hulls," *Applied ocean research,* vol. 42, pp. 105–113, 2013.

[2] A. Trimulyono, M. L. Hakim, C. Ardhan, S. T. P. Ahmad, T. Tuswan, and A. W. B. Santosa, "Analysis of the double steps position effect on planing hull performances," *Brodogradnja: Teorija i praksa brodogradnje i pomorske tehnike,* vol. 74, no. 4, pp. 41–72, 2023.

[3] A. G. Avci and B. Barlas, "Investigation of the optimum longitudinal single transverse step location for a high-speed craft," *Brodogradnja: Teorija i praksa brodogradnje i pomorske tehnike,* vol. 74, no. 3, pp. 47–70, 2023.

[4] S. Samuel, O. Mursid, S. Yulianti, K. Kiryanto, and M. Iqbal, "Evaluation of interceptor design to reduce drag on planing hull," *Brodogradnja: Teorija i praksa brodogradnje i pomorske tehnike,* vol. 73, no. 3, pp. 93–110, 2022.

[5] O. S. Sahin, E. Kahramanoglu, F. Cakici, and E. Pesman, "Control of dynamic trim for planing vessels with interceptors in terms of comfort and minimum drag," *Brodogradnja: Teorija i praksa brodogradnje i pomorske tehnike,* vol. 74, no. 1, pp. 1–17, 2023.

[6] T. Von Karman, "The impact on seaplane floats during Technical Notes," *National Advisory Committee for,* 1929.

[7] H. Wagner, "Über stoß-und gleitvorgänge an der oberfläche von flüssigkeiten," *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik,* vol. 12, no. 4, pp. 193–215, 1932.

[8] E. P. Clement and D. L. Blount, "Resistance tests of a systematic series of planing hull forms," *Trans. sname,* vol. 71, no. 3, pp. 491–579, 1963.

[9] G. Fridsma, "A systematic study of the rough-water performance of planing boats," Stevens Inst of Tech Hoboken NJ Davidson Lab, 1969.

[10] G. Fridsma, "A Systematic Study of the Rough-Water Performance of Planing Boats. Irregular Waves-Part 2," STEVENS INST OF TECH HOBOKEN NJ DAVIDSON LAB, 1971.

[11] T. Katayama, T. Hinami, and Y. Ikeda, "Longitudinal motion of a super high-speed planing craft in regular head waves," in *Proc. 4th Osaka Colloquium on Seakeeping Performance of Ships*, 2000, pp. 214–220.

[12] K. Garme, "Improved time domain simulation of planing hulls in waves by correction of the near-transom lift," *International Shipbuilding Progress,* vol. 52, no. 3, pp. 201–230, 2005.

[13] P. Ghadimi, A. Dashtimanesh, S. R. Djeddi, and Y. F. Maghrebi, "Development of a mathematical model for simultaneous heave, pitch and roll motions of planing vessel in regular waves," *Int. J. Sci. World,* vol. 1, no. 2, pp. 44–56, 2013.

[14] S. Wang, Y. Su, X. Zhang, and J. Yang, "RANSE simulation of high-speed planning craft in regular waves," *Journal of Marine Science and Application,* vol. 11, no. 4, pp. 447–452, 2012.

[15] M. Martin, "Theoretical determination of porpoising instability of high-speed planing boats," *Journal of ship research,* vol. 22, no. 01, pp. 32–53, 1978.

[16] E. E. Zarnick, "A nonlinear mathematical model of motions of a planing boat in regular waves," DAVID W TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER BETHESDA MD, 1978.

[17] T. M. Sayeed, H. Peng, B. Veitch, and R. Billard, "NUMERICAL SIMULATION OF FAST RESCUE CRAFTS IN WAVES AND ITS APPLICATION IN A TRAINING SIMULATOR," *Journal of Ocean Technology,* vol. 8, no. 4, 2013.

[18] J. Keuning, "The nonlinear behaviour of fast monohulls in head waves [Ph. D. thesis]," ed: Technische Universiteit Delft, Delft, The Netherlands, 1994.

[19] A. Van Deyzen, "A nonlinear mathematical model of motions of a planing monohull in head seas," in *Proceedings of the 6th International Conference on High Performance Marine Vehicles (HIPER'08)*, 2008.

[20] F.-c. Chiu and M. Fujino, "Nonlinear prediction of vertical motions and wave loads of high-speed crafts in head sea," *International shipbuilding progress,* vol. 36, no. 406, 1989.

[21] J. D. Hicks, A. W. Troesch, and C. Jiang, "Simulation and nonlinear dynamics analysis of planning hulls," *ASME DYN SYST CONTROL DIV PUBL DSC, ASME, NEW YORK, NY,(USA), 1993,* vol. 51, pp. 41–55, 1993.

[22] J. I. R. Blake, "Investigation into the vertical motions of high speed planing craft in calm water and in waves," University of Southampton, 2000.

[23] S. G. Lewis, D. A. Hudson, S. R. Turnock, J. I. Blake, and R. Shenoi, "Predicting the motions of high speed RIBs: a comparison of non-linear strip theory with experiments," 2006.

[24] R. Niazmand Bilandi, A. Dashtimanesh, and S. Tavakoli, "Development of a 2D+T theory for performance prediction of double-stepped planing hulls in calm water," *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment,* vol. 233, no. 3, pp. 886–904, 2019.

[25] P. Ghadimi, S. Tavakoli, and A. Dashtimanesh, "An analytical procedure for time domain simulation of roll motion of the warped planing hulls," *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment,* vol. 230, no. 4, pp. 600–615, 2016.

[26] S. Tavakoli, P. Ghadimi, and A. Dashtimanesh, "A nonlinear mathematical model for coupled heave, pitch, and roll motions of a high-speed planing hull," *Journal of Engineering Mathematics,* vol. 104, no. 1, pp. 157–194, 2017.

[27] S. Tavakoli and A. Dashtimanesh, "Mathematical simulation of planar motion mechanism test for planing hulls by using 2D+ T theory," *Ocean Engineering,* vol. 169, pp. 651–672, 2018.

[28] S. Hosseinzadeh, M. Khorasanchi, and M. S. Seif, "Investigation of planing vessels motion using nonlinear strip theory–an experimental and numerical study," *Ships and Offshore Structures,* vol. 14, no. 7, pp. 684–697, 2019.