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New mathematical models for the load factor of slip pairs in the ship propulsion system for non-Newtonian lubricants

Oleksandr Kryvyi[1*], Mykhaylo Miyusov[2], Mark Kryvyi[3]

[1] National University "Odessa Maritime Academy", Department of Higher Mathematics, 8, Didrikchson str. 65052, Odesa, Ukraine, e-mail: krivoy-odessa@ukr.net, ORCID: https://orcid.org/0000-0001-8704-4791

[2] National University "Odessa Maritime Academy", 8, Didrikchson str. 65052, Odesa, Ukraine, e-mail: rector@onma.edu.ua, ORCID: https://orcid.org/0000-0001-6430-4677

[3] National University "Odessa Maritime Academy", Department of Ship Power Plants, 8, Didrikchson str. 65052, Odesa, Ukraine, e-mail: markkrivoy1@gmail.com, ORCID: https://orcid.org/0000-0001-7507-7203

* Corresponding author

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ABSTRACT

The method of boundary variation was applied to the solution of the boundary value problem with respect to the Reynolds differential equation for the lubricating layer in the sliding bearings of ship power plants and auxiliary ship equipment, which made it possible to obtain analytical representations for the hydrodynamic pressure, shear stresses, as well as the integral characteristics of the lubricating layer. This made it possible to build new, easy-to-use mathematical models for the load factor (carrying force factor) of the sliding pairs of the ship propulsion complex, considering the non-Newtonian properties of lubricants, i.e. in the case of the dependence of dynamic viscosity on pressure and temperature. Mathematical models take into account geometric parameters of bearings, operational parameters: relative radial clearance and relative eccentricity, angular velocity; as well as viscosity characteristics of lubricants, in particular, dynamic viscosity and piezo coefficient of viscosity of lubricants. A criterion for the presence of an oil layer in the working zone of the sliding pair at given operational characteristics and parameters of the sliding bearing is proposed, which considers the viscous characteristics of lubricants. The research results are illustrated in the form of tables and graphs.

1 Introduction

The ship's propulsion system, which includes power plants and auxiliary ship equipment, is a complex engineering system and consists of many different units and mechanisms (see, in particular, [1]). The durability of the entire engineering ship system depends on the durability of each individual unit, the reliability of each of which is usually calculated separately. One of these important components is sliding pairs (sliding bearings), which are present both in the main power plants and in auxiliary equipment. In particular, these are the main and connecting rod necks of the crankshafts, the propeller shaft bearings, the rudder shaft bearings. Research in this direction was carried out by many authors, particularly the fundamental theory of sliding bearings, which is presented in the work [2], and received further development in the works [3, 4, 5, 6]. A number of works are devoted to the study of the reliability of propeller thrust shaft bearings, in particular, in work [7] various propeller shaft bearings were experimentally investigated with different lubrication methods, in paper [8], with the help of numerical solutions of the Reynolds equation tribological characteristics of a marine stern tube bearing were studied, considering bending deformation of stern shaft and cavitation. In works [9, 10, 11], with the help of analytical solutions of the Reynolds equation, various aspects of the durability of operation of ship support sliding bearings were studied, in work [12] analytical dependences for the angles of the beginning and end of the working zone on the relative eccentricity ε_0 and relative radial clearance δ_0 were obtained in a pair of slips. Works [13, 14] are devoted to the study of operational characteristics, such as efficiency of ship propellers and rudders. One of the main factors that affect the operation of sliding bearings is the viscosity characteristics of lubricants, in work [15] the classification of marine lubricants and their characteristics are given. Studies [16 – 18] show that in connection with extreme operating modes, the behavior of lubricants in the working zone of the sliding pairs of the ship system is non-Newtonian [2, 17] in nature, failure to take it into account leads to significant errors in design and forecasting trouble-free operation. Contamination of lubricants during operation and impurities [19, 20], which are used to improve the viscous characteristics of lubricants, also lead to non-Newtonian behavior of lubricants. The calculation and forecasting of the trouble-free operation of the sliding bearings of the ship propulsion system is usually based on the integral characteristics of the sliding pair, such as, for example, the dimensionless load factor [2 – 5], which is used, in particular, in the criterion Sommerfeld [2, 4, 21].

The existing dependences of the coefficient Φ_P on the relative eccentricity ε_0 of the sliding pair are usually tabular or graphical [2, 3, 21] and obtained for a small number of values ε_0. In addition, the existing representations of the load factor are obtained only for Newtonian lubricants and do not take into account their dynamic viscosity, which, taking into account the above, can lead to significant errors when calculating the sliding bearings of the ship's propulsion system.

The purpose of this work is to study the influence of non-Newtonian behavior of lubricants on the load factor Φ_P of a sliding bearing and to build new adequate mathematical models for Φ_P, which would be applicable for any geometric and operational characteristics of sliding bearings and take into account the viscous characteristics of lubricants.

2 The formulation and mathematical model of the problem

Let the sliding bearing of length L, the radius of the sleeve R_2, and the radius of the shaft (trunnion) R_1 be loaded by an external force: $\vec{F} = (P;0)$. This force in the stable mode of operation of the sliding pair, that is, when the trunnion rotates at a constant speed $\omega_0[c^{-1}]$, will be balanced by hydrodynamic forces in the lubricating layer of the sliding pair. At the same time, there is a representation [2, 21]

$$P = 2R_1 L k_1 \Phi_P, \quad (1)$$

where $k_1 = \frac{\mu_0\omega_0}{\delta_0^2}\left[\frac{kg}{m\cdot c^2} = 1\text{Pa}\right]$, $\delta_0 = \frac{\delta}{R_1}$ the relative radial gap of the sliding bearing; $\delta = (R_2 - R_1)$ – the radial gap of the sliding bearing; $\mu_0\left[\frac{N\cdot c}{m^2}\right]$ the dynamic viscosity of lubricants at atmospheric pressure. The load factor for a finite bearing Φ_P is related to the load factor of an infinite bearing Φ_P^∞ as

$$\frac{\Phi_P}{\Phi_P^\infty} = \kappa_1 \quad (2)$$

where the dimensionless coefficient κ_1 can be determined in different ways [2].

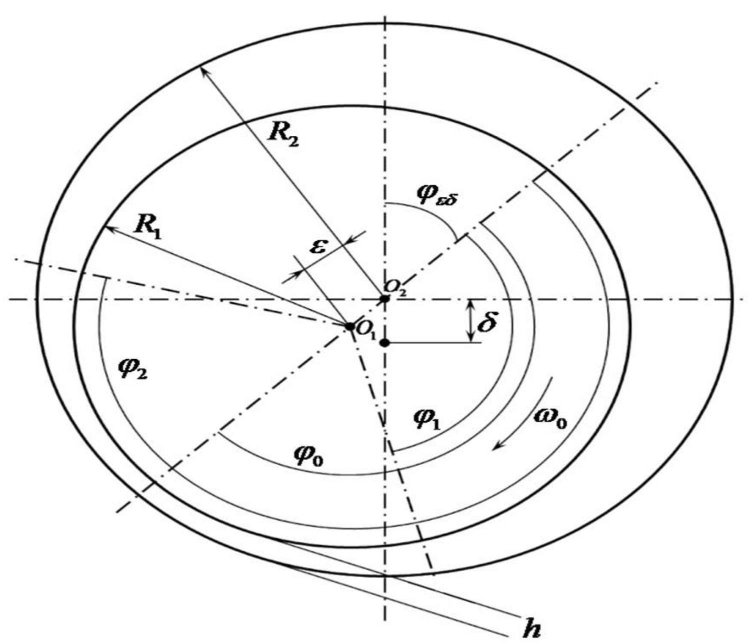

Figure 1 Slip pair model

Therefore, to determine the load factor, it is sufficient to determine the load factor of an infinite bearing Φ_p^∞, that is, a bearing in which there is no leakage of lubricants at the ends, the so-called flat problem of lubrication theory (see Fig. 1). This problem is formulated in the form of a boundary value problem for the Reynolds differential equation [2, 3, 4, 5] with respect to the specific pressure distribution in the lubricating layer $\tilde{p}(\varphi) = k_1^{-1}p(\varphi)$, where $p(\varphi)$ – the pressure in the lubricating layer, $\varphi \in (\varphi_1, \varphi_2)$, φ_1, φ_2 – the beginning and end of the working zone of the lubricating layer. For non-Newtonian lubricants, dynamic viscosity depends on pressure and is determined using the Barus formula [2, 7]:

$$\mu = \mu_0 e^{\xi p(\varphi)} = \mu_0 e^{\tilde{\xi}\tilde{p}(\varphi)},\ \tilde{\xi} = \xi \cdot k_1, \tag{3}$$

where where ξ [Pa^{-1}] is the piezo coefficient of viscosity of lubricants, μ_0 – dynamic viscosity at atmospheric pressure. Both parameters are determined experimentally and depend on temperature [5, 17, 18].

After introducing a new unknown function:

$$q(\varphi) = e^{-\tilde{\xi}\cdot\tilde{p}(\varphi)}),\ \tilde{p}(\varphi) = -\tilde{\xi}^{-1}\ln q(\varphi), \tag{4}$$

we present the specified boundary value problem as follows:

$$\frac{d}{d\varphi}\left(\tilde{h}^3\frac{dq}{d\varphi}\right) = -6\tilde{\xi}\frac{d\tilde{h}}{d\varphi},\ \varphi_1 < \varphi < \varphi_2, \tag{5}$$

$$q(\varphi_1) = q(\varphi_2) = 1;\ q'(\varphi_2) = 0, \tag{6}$$

where $\tilde{h}(\varphi) = \delta^{-1} h(\varphi) = 1 + \varepsilon_0 \cos\varphi$, $h(\varphi)$ – the thickness of the lubricating layer.

3 Determination of the specific pressure in the lubricating layer and the applicability criterion of the model for non-Newtonian lubricants

From equation (5), after integration, we get

$$\frac{dq}{d\varphi} = -6\tilde{\xi}\frac{\tilde{h}+C}{\tilde{h}^3}. \tag{7}$$

Taking into account the first two conditions from (6) and Rolle's theorem (see for example [22]), we can claim that there is a point φ_0 on the interval (φ_1, φ_2), where $q'(\varphi_0) = 0$. The last equality makes it possible to present the unknown C as $C = -\tilde{h}_0 = -1 - \varepsilon_0\cos(\varphi_0)$, and to write the differential equation in the form

$$dq = -6\tilde{\xi}\frac{\tilde{h}(\varphi)-\tilde{h}_0}{\tilde{h}^3(\varphi)}d\varphi. \tag{8}$$

It should be noted that at the point φ_0 the function $q(\varphi)$ reaches a minimum, and the relative hydrodynamic pressure $\tilde{p}(\varphi)$ reaches a maximum.

The exact solution of the differential equation (8), which satisfies the second boundary condition (6), is given as

$$\begin{aligned} q(\varphi) &= 1 + 6\tilde{\xi}\int_\varphi^{\varphi_2}\frac{\tilde{h}(\psi)-\tilde{h}_0}{\tilde{h}^3(\psi)}d\psi = \\ &= 1 + 6\tilde{\xi}\varepsilon_0\int_\varphi^{\varphi_2}\frac{\cos\psi - \cos\varphi_0}{\tilde{h}^3(\psi)}d\psi. \end{aligned} \tag{9}$$

Having implemented the first boundary condition from (5), we obtain a condition for determining the angle φ_0, i.e.

$$q(\varphi_1) = 1;\ \Rightarrow \int_{\varphi_1}^{\varphi_2}\frac{\cos\psi - \cos\varphi_0}{(1+\varepsilon_0\cos\psi)^3}d\psi = 0. \tag{10}$$

Next, using the second formula (4), we will find the distribution of the relative specific hydrodynamic pressure $\tilde{p}(\varphi)$ in the working layer of the sliding pair for a non-Newtonian lubricant

$$\tilde{p}(\varphi) = \frac{-1}{\tilde{\xi}}ln(q(\varphi)) = \frac{-1}{\tilde{\xi}}ln\left(1 + 6\tilde{\xi}\int_\varphi^{\varphi_2}\frac{\tilde{h}(\varphi)-\tilde{h}_0}{\tilde{h}^3(\varphi)}d\psi\right). \tag{11}$$

The obtained distribution of the specific pressure of the lubricating layer for non-Newtonian lubricants, in addition to the relative eccentricity ε_0, also contains a dimensionless parameter $\tilde{\xi} = \frac{\xi\mu_0\omega_0}{\delta_0^2} \geq 0$. Note that for $\tilde{\xi} \to 0$ the obtained solution (11) leads to the distribution of the relative specific hydrodynamic pressure for Newtonian lubricant.

The existence of a solution of the boundary value problem (5), (6) and the presence of a lubricating layer in the sliding pair depends on the value of the parameter $\tilde{\xi}$. Indeed, the obtained solution (9) can be considered correct only if the condition $0 < q(\varphi) \leq 1$ is fulfilled, which leads to the necessity of fulfilling the following condition:

$$0 \leq \tilde{q}_0(\varphi, \varepsilon_0) < \frac{1}{\tilde{\xi}}, \tag{12}$$

$$\tilde{q}_0(\varphi, \varepsilon_0) = \frac{1}{\tilde{\xi}}(1 - q(\varphi)) = 6\int_\varphi^{\varphi_2}\frac{\cos\varphi_0 - \cos\psi}{\tilde{h}^3(\psi)}d\psi.$$

Given that the function $q(\varphi)$ at the point φ_0 reaches a minimum, and at the points φ_1, φ_2 – a maximum, moreover $q(\varphi_1) = q(\varphi_2) = 1$, then the function $\tilde{q}_0(\varphi, \varepsilon_0)$ at the points φ_1, φ_2 will reach a minimum, moreover $\tilde{q}_0(\varphi_1, \varepsilon_0) = \tilde{q}_0(\varphi_2, \varepsilon_0) = 0$, and at the point φ_0 will reach a maximum: $q_0(\varepsilon_0) = \tilde{q}_0(\varphi_0, \varepsilon_0) = \max\limits_{\varphi_1\leq\varphi\leq\varphi_2}\tilde{q}_0(\varphi, \varepsilon_0)$. Therefore, the condition for the applicability of lubricants for the specified modes of operation of the sliding pair is the fulfillment of the criterion:

$$\tilde{\xi} < K_\mu,\ K_\mu = (q_0(\varepsilon_0))^{-1}. \tag{13}$$

Table 1 Criterion values for some values of relative eccentricity ε_0

ε_0	0.01	0.1	0.3	0.4	0.5	0.6	0.7	0.75	0.80	0.90	0.95	0.99
K_μ	14.18	1.48	0.57	0.396	0.28	0.20	0.13	0.10	0.07	$2.7\cdot10^{-2}$	$9.9\cdot10^{-3}$	$9.2\cdot10^{-4}$

Table 2 The maximum possible values of the viscosity gradient $G^*_{\mu0}\cdot10^8$ at $\varepsilon_0 = 0.5$

$\delta_0\cdot10^3$	ω_0 [1/s (rpm)]							
	3.14 (30)	**6.28 (60)**	**8.34 (80)**	**10.47 (100)**	**12.57 (120)**	**14.66 (140)**	**18.85 (180)**	**28.27 (270)**
0.93	7.82	3.91	2.94	2.35	1.95	1.68	1.3	0.87
1.0	9.04	4.52	3.40	2.71	2.26	1.94	1.51	1.0
1.23	13.68	6.84	5.15	4.10	3.41	2.93	2.28	1.52
1.63	24.02	12.01	9.04	7.20	6.00	5.14	4.00	2.67
2.0	36.16	18.08	13.61	10.84	9.03	7.74	6.02	4.02
2.64	63.00	31.99	23.72	18.89	15.74	13.49	10.49	7.00
3.0	81.35	40.07	30.63	24.40	20.32	17.43	13.55	9.04
3.5	110.73	55.36	41.69	33.21	57.66	23.72	18.45	12.30
5.0	225.97	112.99	85.08	67.77	56.49	48.40	37.64	25.10

Table 1 shows values K_μ for some values of relative eccentricity ε_0, which show that the criterion K_μ satisfies the condition

$$9.2\cdot10^{-4} \le K_\mu < 15, \tag{14}$$

Criterion (13) makes it possible to evaluate the correctness of the use of specific non-Newtonian oils at the given technical parameters of the bearing and the given speed of rotation of the trunnion. This means that not considering the dependence of dynamic viscosity on pressure and temperature can distort the forecast of trouble-free operation of the sliding pair.

The viscosity behavior of lubricants when the pressure in the sliding bearing changes is determined by the viscosity gradient [2]: $G_\mu(p) = \frac{d\mu}{dp}$, which, according to formula (3), can be written as $G_\mu(p) = \xi\mu_0\cdot e^{\xi p(\varphi)}$. It follows that the viscosity gradient at atmospheric pressure is equal to $G_{\mu0} = \xi\mu_0$. On the other hand, from the representation $\tilde{\xi} = \frac{\xi\mu_0\omega_0}{\delta_0^2}$, we find $\xi\mu_0 = \frac{\tilde{\xi}\delta_0^2}{\omega_0}$, this makes it possible to present the criterion (13) as follows

$$0 \le G_{\mu0} < \frac{K_\mu\delta_0^2}{\omega_0} \tag{15}$$

Conditions (15) make it possible to determine the maximum possible value of the viscosity gradient $G^*_{\mu0}$, that is, the maximum possible values of the product $\xi\mu_0$, for different operating modes of the sliding pair in which the oil layer is preserved, in particular, table 2 shows such values for sliding pairs of low- and medium-speed engines at relative eccentricity $\varepsilon_0 = 0.5$.

With the help of Table 2, it is possible to determine whether the viscosity parameters of the selected lubricant are suitable for the specified operating modes of the sliding bearing of the ship's propulsion system. Similar tables can be obtained for other values of relative eccentricity ε_0 under different operating modes. It should be noted that since for Newtonian lubricants $\tilde{\xi} = 0$, inequality (13) will always hold for Newtonian lubricants.

4 Determination of the distribution of viscous shear stresses in the lubricating layer for non-Newtonian lubricants

Shear stresses in the lubricating layer in the plane theory of lubrication allow the representation [2]

$$\tau_\varphi = \mu\frac{\omega_0 R_1}{h} + \frac{1}{2R_1}\frac{dp}{d\varphi}(2y - h) \tag{16}$$

Using the relations:

$p_n(\varphi) = k_1\tilde{p}_n(\varphi)$, $\mu(p) = \frac{\mu_0}{e^{-\tilde{\xi}\tilde{p}_n(\varphi)}} = \frac{\mu_0}{q(\varphi)}$, and entering the notations $k_3 = \frac{\mu_0\omega_0}{\delta_0}$, $\tilde{y} = \frac{y}{\delta}$, we will move to dimensionless quantities in representation (16)

$$\tilde{\tau}_\varphi = \frac{\tau_\varphi}{k_3} = \frac{1}{\tilde{h}\cdot q(\varphi)} + \frac{1}{2}\frac{d\tilde{p}}{d\varphi}(2\tilde{y} - \tilde{h}) \tag{17}$$

Considering the formula $\tilde{p}_n'(\varphi) = \frac{-1}{\tilde{\xi}}\frac{q'(\varphi)}{q(\varphi)}$, representation (17) will be rewritten as follows:

$$\tilde{\tau}_\varphi = \frac{1}{q(\varphi)}\left(\frac{1}{\tilde{h}} - \frac{1}{2\tilde{\xi}}q'(\varphi)(2\tilde{y} - \tilde{h})\right).$$

Then, using formula (8), we get

$$\tilde{\tau}_\varphi = \frac{1}{q(\varphi)}\left(\frac{1}{\tilde{h}} + 3\frac{\tilde{h}(\varphi)-\tilde{h}_0}{\tilde{h}^3(\varphi)}(2\tilde{y}-\tilde{h})\right) \quad (18)$$

Shear stresses for non-Newtonian lubricants, according to formula (18), vary with the thickness of the lubricating layer. In particular, on the trunnion, with $\tilde{y} = \tilde{h}$, we get:

$$\tilde{\tau}_{\varphi h} = \tilde{\tau}_\varphi\Big|_{\tilde{y}=\tilde{h}} = \frac{4\tilde{h}(\varphi)-3\tilde{h}_0}{q(\varphi)\tilde{h}^2(\varphi)}. \quad (19)$$

5 Determination of the load factor and the angle of the center line for non-Newtonian lubricants

In the projections on the *X, Y* axis, the balance of forces [2, 3, 4, 5] on the sliding bearing of length *L* in the absence of end leakage of lubricants is presented as follows (see Fig. 1)

$$P = -LR_1\int_{\varphi_1}^{\varphi_2} cos(\pi-(\psi+\varphi_{\varepsilon\delta}))p(\psi)d\psi - \\ - LR_1\int_{\varphi_1}^{\varphi_2} sin(\pi-(\psi+\varphi_{\varepsilon\delta}))\tau_\varphi(\psi)d\psi. \quad (20)$$

$$0 = LR_1\int_{\varphi_1}^{\varphi_2} sin(\pi-(\psi+\varphi_{\varepsilon\delta}))p(\psi)d\psi - \\ - LR_1\int_{\varphi_1}^{\varphi_2} cos(\pi-(\psi+\varphi_{\varepsilon\delta}))\tau_\varphi(\psi)d\psi. \quad (21)$$

In (20), (21) let us pass to dimensionless quantities, then, using the fact that $k_3 = \delta_0 k_1$, and formula (1), we write

$$2\Phi_P^\infty = \int_{\varphi_1}^{\varphi_2} cos(\psi+\varphi_{\varepsilon\delta})\tilde{p}(\psi)d\psi - \\ - \delta_0\int_{\varphi_1}^{\varphi_2} sin(\psi+\varphi_{\varepsilon\delta})\tilde{\tau}_\varphi(\psi)d\psi. \quad (22)$$

$$0 = \int_{\varphi_1}^{\varphi_2} sin(\psi+\varphi_{\varepsilon\delta})\tilde{p}(\psi)d\psi - \\ - \delta_0\int_{\varphi_1}^{\varphi_2} cos(\psi+\varphi_{\varepsilon\delta})\tilde{\tau}_\varphi(\psi)d\psi. \quad (23)$$

We will perform the integration by parts in the first integrals of the last equations, using the fact that $\tilde{p}(\varphi_1) = \tilde{p}(\varphi_2) = 0$, as a result, we will get

$$\Phi_P^\infty = -\frac{1}{2}\int_{\varphi_1}^{\varphi_2} sin(\psi+\varphi_{\varepsilon\delta})(\tilde{p}'(\psi)+\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi. \quad (24)$$

$$0 = \int_{\varphi_1}^{\varphi_2} cos(\psi+\varphi_{\varepsilon\delta})(\tilde{p}'(\psi)-\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi. \quad (25)$$

Having applied the trigonometric formulas for the addition of angles for sine and cosine, relations (24) and (25), we rewrite as follows

$$\Phi_P^\infty = \frac{-1}{2} sin\,\varphi_{\varepsilon\delta}\int_{\varphi_1}^{\varphi_2} cos\,\psi\cdot(\tilde{p}'(\psi)+\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi + \\ + \frac{-1}{2} cos\,\varphi_{\varepsilon\delta}\int_{\varphi_1}^{\varphi_2} sin\,\psi\cdot(\tilde{p}'(\psi)+\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi. \quad (26)$$

$$0 = cos\,\varphi_{\varepsilon\delta}\int_{\varphi_1}^{\varphi_2} cos\,\psi\,(\tilde{p}'(\psi)-\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi - \\ - sin\,\varphi_{\varepsilon\delta}\int_{\varphi_1}^{\varphi_2} sin\,\psi\,(\tilde{p}'(\psi)-\delta_0\tilde{\tau}_{\varphi h}(\psi))d\psi. \quad (27)$$

Using representations (9) and (17), we obtain the following formula for calculating the load factor

$$\Phi_P^\infty = A_c^+ \sin\varphi_{\varepsilon\delta} + A_s^+ \cos\varphi_{\varepsilon\delta}. \quad (28)$$

The following notations are introduced here

$A_c^\pm = \pm 2\delta_0 j_1^c + (3 \mp 1.5\tilde{h}_0\delta_0)j_2^c - 3\tilde{h}_0 j_3^c$;
$A_s^\pm = \pm 2\delta_0 j_1^s + (3 \mp 1.5\tilde{h}_0\delta_0)j_2^s - 3\tilde{h}_0 j_3^s$.

$$j_k^c = \int_{\varphi_1}^{\varphi_2}\frac{cos\,\varphi d\varphi}{(1+\varepsilon_0\,cos\,\varphi)^k},\; j_k^s = \int_{\varphi_1}^{\varphi_2}\frac{sin\,\varphi d\varphi}{(1+\varepsilon_0\,cos\,\varphi)^k}.$$

The given representations contain the angle $\varphi_{\varepsilon\delta}$ of deviation of the center line from the vertical axis, for its determination we use the relation (27), which we rewrite as follows

$$A_s^- \sin\varphi_{\varepsilon\delta} = A_c^- \cos\varphi_{\varepsilon\delta}. \quad (29)$$

From here, we get the following equation to determine the angle $\varphi_{\varepsilon\delta}$

$$\text{tg}\varphi_{\varepsilon\delta} = \frac{A_c^-}{A_s^-}. \quad (30)$$

From relation (30) the following representations follow $\frac{A_c^-}{\sqrt{(A_s^-)^2+(A_c^-)^2}}$, $cos\,\varphi_{\varepsilon\delta} = \frac{A_s^-}{\sqrt{(A_s^-)^2+(A_c^-)^2}}$, then formula (28) will be rewritten as follows

$$\Phi_P^\infty(\varepsilon_0,\tilde{\xi}) = \frac{A_c^+A_c^- + A_s^+A_s^-}{\sqrt{(A_s^-)^2+(A_c^-)^2}}. \quad (31)$$

Note that the load factor (31) depends both on the relative eccentricity ε_0, and on the dimensionless parameter $\tilde{\xi} = \frac{\xi\mu_0\omega_0}{\delta_0^2}$, which depends on the viscous characteristics of the lubricants and on the modes of operation of the sliding pair, that is, it is a function of two variables $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$, while $\varepsilon_0 \in (0;1)$, $\tilde{\xi} \in [0; K_\mu)$. For non-Newtonian lubricants $\tilde{\xi} = 0$, at the moment, it is precisely for such a case that the values of the load coefficient are known [2-5], which are obtained in the form of tables. A further task is to obtain, in a form convenient for use, the value of the load factor $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$ for non-Newtonian lubricants.

6 Numerical modeling of the operation of the sliding pair for non-Newtonian lubricants and construction of mathematical models of the load factor

The results obtained above make it possible to carry out a numerical simulation of the operation of the sliding pair, and to determine the load factor of the lubricating layer for non-Newtonian lubricants.

The difficulty of numerical modeling of hydrodynamic processes in a sliding pair is that the boundaries of the working zone of the lubricating layer φ_1, φ_2 are not known in advance. To overcome this problem, it is proposed to apply the method of successive approximation or the method of boundary variation, which consists in the following. By varying the parameters φ_1, φ_2, we will achieve the fulfillment of the following conditions that follow from the proposed mathematical model:

$$\begin{cases} \tilde{p}(\varphi_1) = \tilde{p}(\varphi_2) = 0, \\ \tilde{p}'(\varphi_2) = 0, \\ \varphi_2 + \varphi_0 = 360°. \end{cases} \qquad (32)$$

Note that the third condition in (32) is valid for a half-sliding bearing, in which the supply of lubricants occurs at the point of connection of the liners, i.e., perpendicular to the vertical axis *Y*. It should also be noted that when applying the boundary variation method, we will obtain approximate values of the parameters φ_1, φ_2 with any specified accuracy, and instead of conditions (32), we will use the conditions

$$\begin{cases} |\tilde{p}(\varphi_1)| < e_1, \tilde{p}(\varphi_2) = 0, \\ |\tilde{p}'(\varphi_2)| < e_2, \\ |\varphi_2 + \varphi_0 - 360°| < e_3. \end{cases} \qquad (33)$$

Here e_k, $(k = \overline{1,3})$ – the accuracy of the calculations, while for practical calculations, it is enough to fulfill the conditions $e_1 < 10^{-10}$, $e_2 < 10^{-5}$, $e_3 < 10^{-4}$. The fulfillment of the condition $\tilde{p}(\varphi_2) = 0$ is achieved by choosing the representation of the solution of the differential equation (8).

We will present some results of the implementation of the proposed numerical modeling approach. In particular, the research results showed that the properties of the lubricant do not affect the characteristic angles φ_1, φ_2, φ_0, but significantly affect the load factor. Table 3 shows the values of the angles φ_1, φ_2, φ_0 and values of the load factor $\Phi_P^\infty(\varepsilon_0, 0)$, and its logarithm $\ln \Phi_P^\infty(\varepsilon_0, 0)$ at $\tilde{\xi} = 0$, for different values of the relative eccentricity ε_0.

Based on the obtained data (Table 3), we will build a mathematical model of the load factor for Newtonian lubricants $\Phi_P^\infty(\varepsilon_0, 0)$. Considering the significant difference in the values of the coefficient $\Phi_P^\infty(\varepsilon_0, 0)$ when the relative eccentricity ε_0 changes from zero to one, it is advisable to first switch to the logarithmic scale of values,

Table 3 Values of characteristic angles φ_1, φ_2, φ_0, $\Phi_P^\infty(\varepsilon_0, 0)$, and $\ln \Phi_P^\infty(\varepsilon_0, 0)$

$\boldsymbol{\varepsilon_0}$	$\boldsymbol{\varphi_1}$	$\boldsymbol{\varphi_2}$	$\boldsymbol{\varphi_0}$	$\boldsymbol{\Phi_P^\infty(\varepsilon_0, 0)}$	$\boldsymbol{\ln \Phi_P^\infty(\varepsilon_0, 0)}$
0.001	11.8	254.2	106.09	0.005119448	-5.2747087
0.01	27.7	248.1	111.89	0.047132780	-3.0547866
0.05	30.71	244.34	115.46	0.226615030	-1.4845026
0.1	32.07	241	118.998	0.445096540	-0.8094641
0.2	34.53	234.34	125.65	0.871441053	-0.1376071
0.3	37.12	228.07	131.93	1.302500647	0.264286
0.4	39.96	222.08	137.92	1.769230085	0.5705445
0.5	43.15	216.34	143.66	2.321115383	0.8420478
0.6	46.8	210.75	149.25	3.051276333	1.11556
0.65	48.87	207.98	152.02	3.539085732	1.2638684
0.7	51.14	205.193	154.81	4.168138110	1.4274694
0.75	53.68	202.37	15763	5.026191142	1.6146625
0.8	56.61	199.449	160.55	6.287959295	1.8386366
0.85	60.08	196.37	163.63	8.362471388	2.123754
0.9	64.45	192.981	167.02	12.473578036	2.5236127
0.925	67.22	191.075	168.93	16.567325438	2.8074324
0.95	70.69	188.9043	171.10	24.739155358	3.2083872
0.975	75.59	186.1956	173.80	49.224964865	3.8964009
0.99	80.38	183.87853	176.12	122.65244716	4.8093547

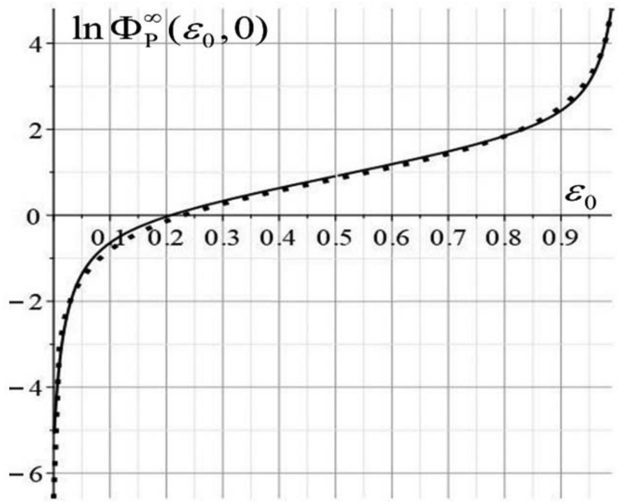

Figure 2 The value of the logarithm of the load factor $\ln(\Phi_P^\infty(\varepsilon_0, 0))$

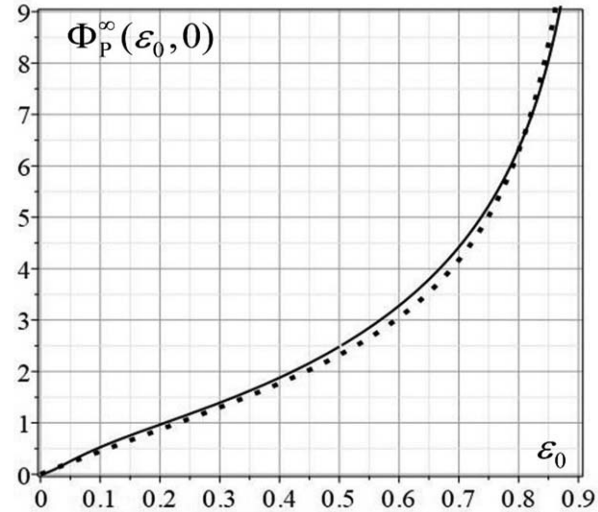

Figure 3 The value of the load factor $\Phi_P^\infty(\varepsilon_0, 0)$

that is, to first conduct simulations for the logarithm of the load coefficient $\ln \Phi_P^\infty(\varepsilon_0, 0)$. So, using the data in Table 3, with the help of regression analysis [23, 24], we will get the following representations

$$\ln \Phi_P^\infty(\varepsilon_0, 0) = \\ = 1.8229\varepsilon_0 - 0.3094 \operatorname{ctg}(3.01287\varepsilon_0 - 3.0838). \tag{34}$$

$$\Phi_P^\infty(\varepsilon_0, 0) = e^{1.8229\varepsilon_0 - 0.3094 \operatorname{ctg}(3.01287\varepsilon_0 - 3.0838)}. \tag{35}$$

Figures 2 and 3 show the dependence of the logarithm of the load factor and the load factor on the relative eccentricity ε_0, which are obtained using dependencies (34), (35) – solid curves in both figures, and linear spline approximation of the data in Table 3 – dotted lines. So, we have an almost complete coincidence of the results, which confirms the adequacy of the obtained mathematical models (34), (35).

Using formula (31), let's analyze how the value of the load factor $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$ depends on the viscosity characteristics of lubricants and the modes of operation of the sliding bearing. To do this, we will obtain graphical dependences Φ_P^∞ of the parameter $\tilde{\xi}$ at different values of relative eccentricity ε_0, in particular, such dependences are shown in Figures 4 – 11, respectively, for the values of $\varepsilon_0 = 0.05$, $\varepsilon_0 = 0.1$, $\varepsilon_0 = 0.3$, $\varepsilon_0 = 0.5$, $\varepsilon_0 = 0.7$, $\varepsilon_0 = 0.8$, $\varepsilon_0 = 0.9$, $\varepsilon_0 = 0.95$.

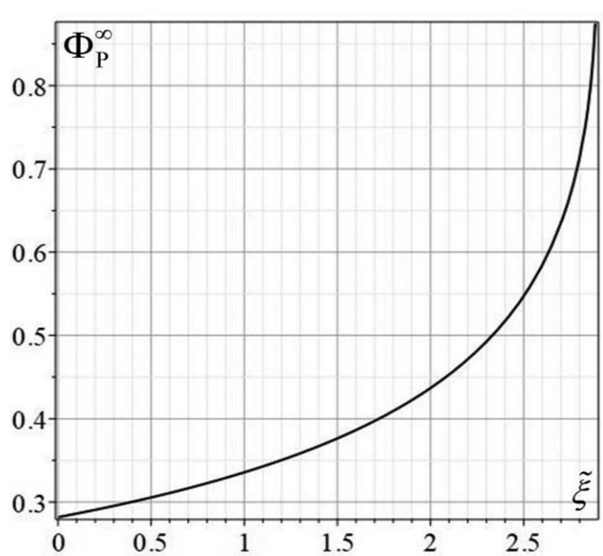

Figure 4 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.05$

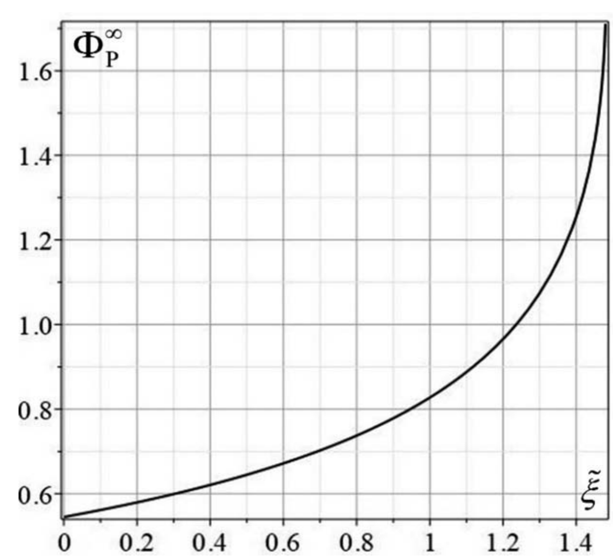

Figure 5 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.1$

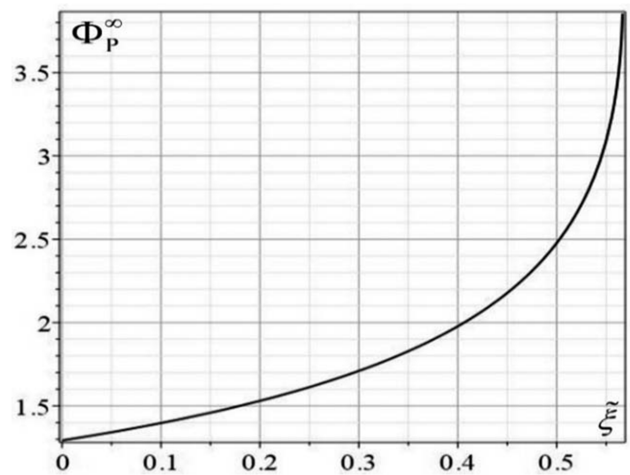

Figure 6 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.3$

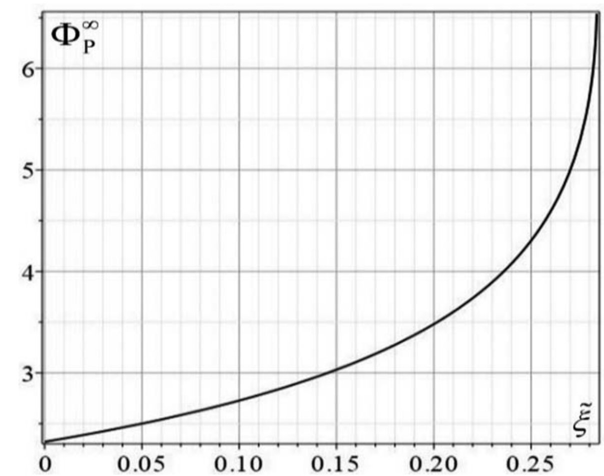

Figure 7 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.5$

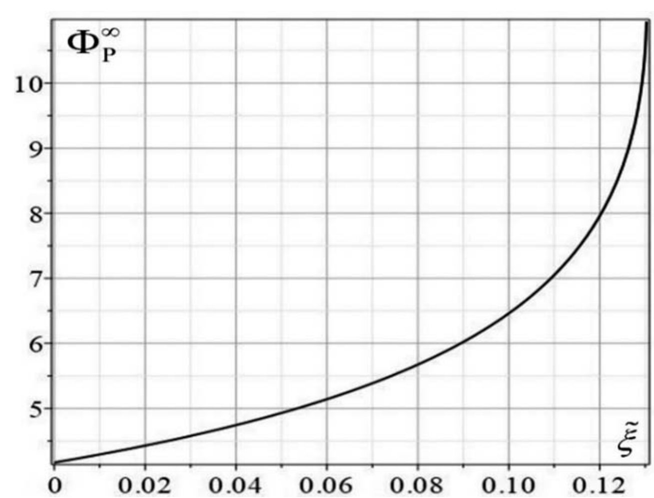

Figure 8 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.7$

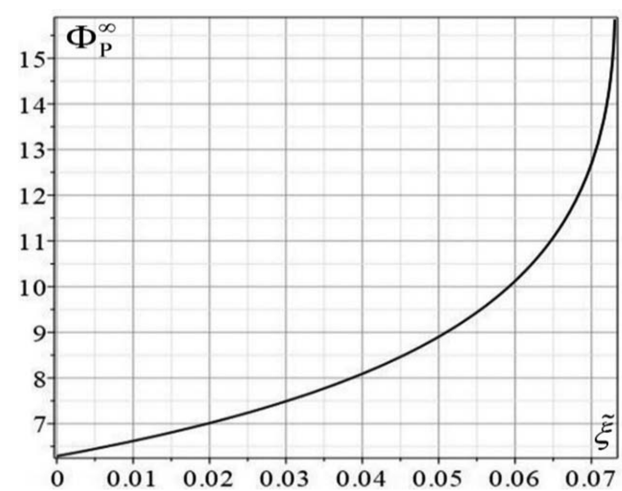

Figure 9 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.8$

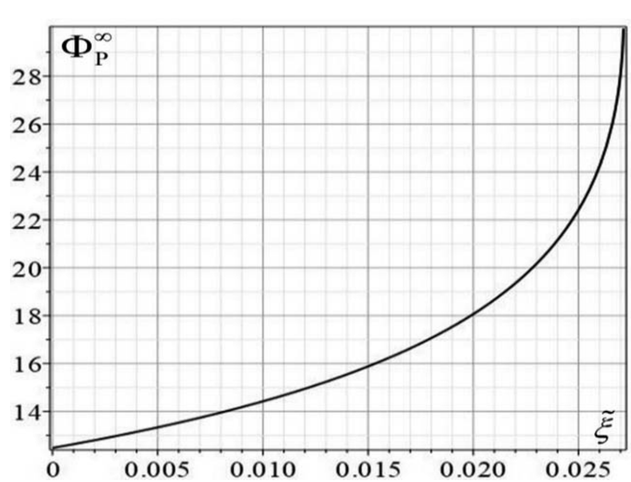

Figure 10 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.9$

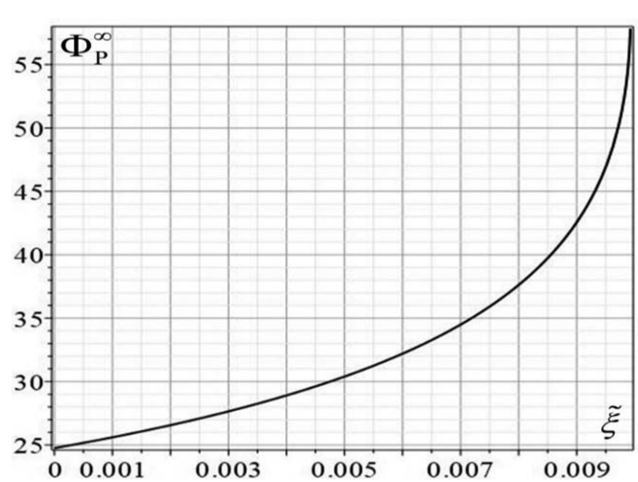

Figure 11 Dependence of the load factor Φ_P^∞ on $\tilde{\xi}$ at $\varepsilon_0 = 0.95$

Table 4 Coefficients of the mathematical model (37)

ε_0	q_0	q_2	ln q_2
0.01	-3.5411150	0.0068989	-4.9763933
0.05	-1.9268959	0.1602953	-1.83073754
0.075	-1.6070517	0.3843729	-0.956142104
0.1	-1.3027678	0.6349967	-0.454135477
0.2	-0.7150877	1.9915983	0.688937482
0.3	-0.4290377	4.4340544	1.48931438
0.4	-0.0672397	8.7571866	2.169874689
0.5	0.2436698	16.8462948	2.824130739
0.6	0.5877585	33.0865478	3.499126789
0.7	0.8695756	86.8106042	4.463728782
0.75	1.2387295	124.3104544	4.822782101
0.8	1.4806932	249.5533835	5.519672854
0.825	1.6584834	363.046061	5.894529716
0.85	1.9309120	512.3755801	6.239057911
0.9	2.3932262	1829.0499931	7.511551982
0.925	2.9030019	3573.0125801	8.181164379
0.95	3.4144470	12644.425962	9.444971762
0.975	4.4663414	88762.1098311	11.39371515

The graphs of the load factor versus the parameter $\tilde{\xi}$ for different values of the relative eccentricity ε_0 shown in Figures 4-11 show a significant influence of the viscous properties of lubricants on $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$. The increase in the values of the latter can exceed 200 % compared to Newtonian lubricants at all values ε_0, while even an insignificant increase in the parameter $\tilde{\xi}$ relative to zero leads to a significant (up to 30%) increase in the values of the load factor, which confirms the need to take into account the viscous characteristics of lubricants when determining Φ_P^∞.

The calculation of the values of the load factor according to formula (29) is associated with the need to sequentially calculate several integrals that at $\varepsilon_0 \to 1$ approach improper integrals, which leads to significant problems in their calculation and complicates their application. To avoid this problem, we will build suitable mathematical models for the load factor

To build mathematical models for $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$, which would consider both parameters ε_0 and $\tilde{\xi}$, and would be easy to apply, we will present the load factor as

$$\Phi_P^\infty(\varepsilon_0, \tilde{\xi}) = \Phi_P^\infty(\varepsilon_0, 0) \cdot Q(\varepsilon_0, \tilde{\xi}). \quad (36)$$

For the load factor for Newtonian lubricants $\Phi_P^\infty(\varepsilon_0, 0)$, we use representation (35), for the function $Q(\varepsilon_0, \tilde{\xi})$, taking into account the condition $Q(\varepsilon_0, 0) = 1$, with the help of regression analysis methods [23, 24], the following dependence is obtained

$$Q(\varepsilon_0, \tilde{\xi}) = 1 + \tilde{\xi} e^{q_2 \tilde{\xi}^2 + q_0}. \quad (37)$$

Table 4 shows the values for the coefficients in representation (37) for different values of the relative eccentricity ε_0.

Given the nonlinearity of the representation (37) with respect to $\tilde{\xi}$, let's check the adequacy of these models. Figures 12-27 show the dependences for the function $Q(\varepsilon_0, \tilde{\xi})$ from $\tilde{\xi}$ which the solid lines are obtained using the mathematical model (37), and the spline approximation of the data obtained using the formula (31) are the dotted lines, respectively for the values of $\varepsilon_0 = 0.01$, $\varepsilon_0 = 0.1$, $\varepsilon_0 = 0.3$, $\varepsilon_0 = 0.5$, $\varepsilon_0 = 0.7$, $\varepsilon_0 = 0.8$, $\varepsilon_0 = 0.9$, $\varepsilon_0 = 0.975$.

In all figures, the graphs are almost identical, which indicates the adequacy of the obtained mathematical model (37) for all values of ε_0.

To determine the coefficients of mathematical models (37) for any values of relative eccentricity ε_0, using the data in Table 4, regression analysis methods [23, 24], we obtain the following representations

$$q_0 = 0.67187\varepsilon_0 - 0.751011 \cdot \mathrm{ctg}(2.83199\varepsilon_0 - 2.95319). \quad (38)$$

$$q_2 = e^{4.40042\sqrt{\varepsilon_0} - 1.17926 \cdot \mathrm{ctg}(2.84648\varepsilon_0 - 2.93861)}. \quad (39)$$

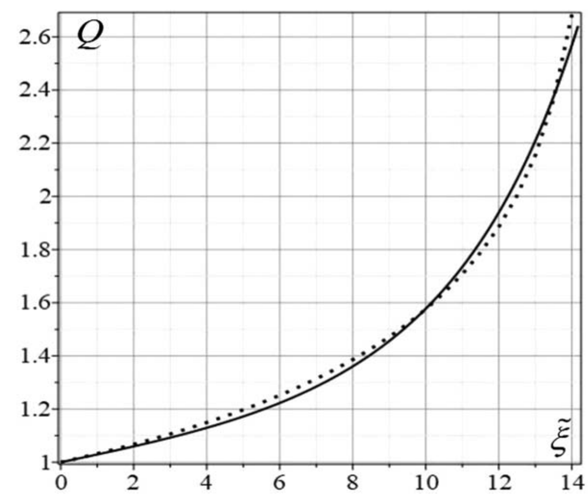

Figure 12 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.01$

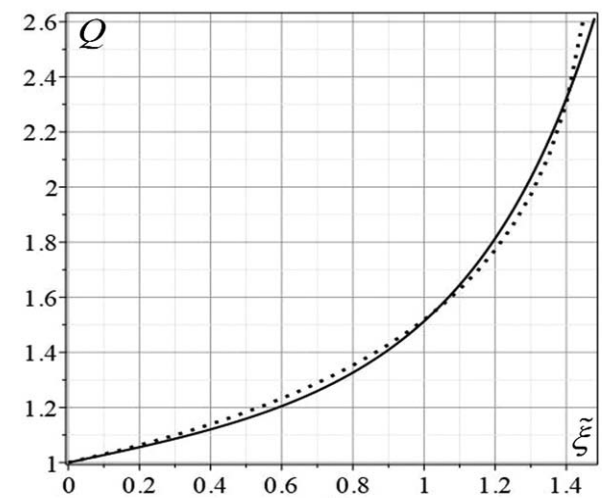

Figure 13 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.1$

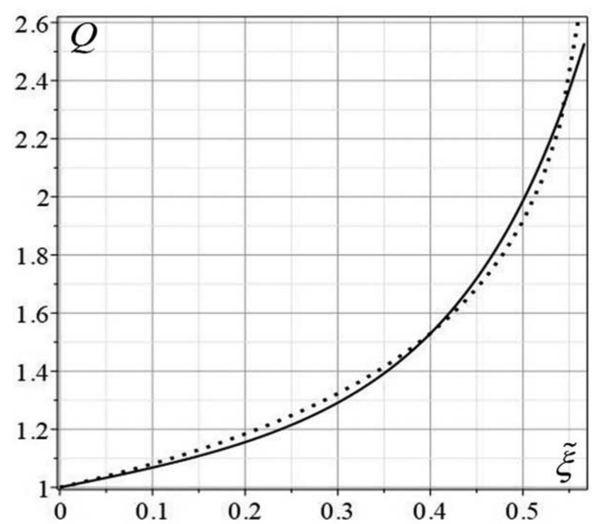

Figure 14 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.3$

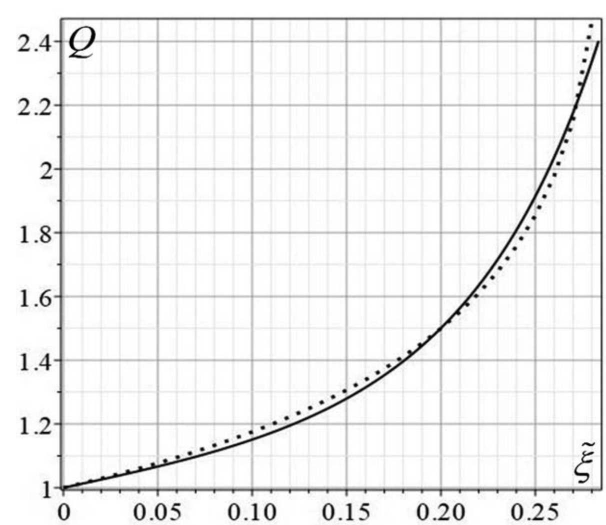

Figure 15 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.5$

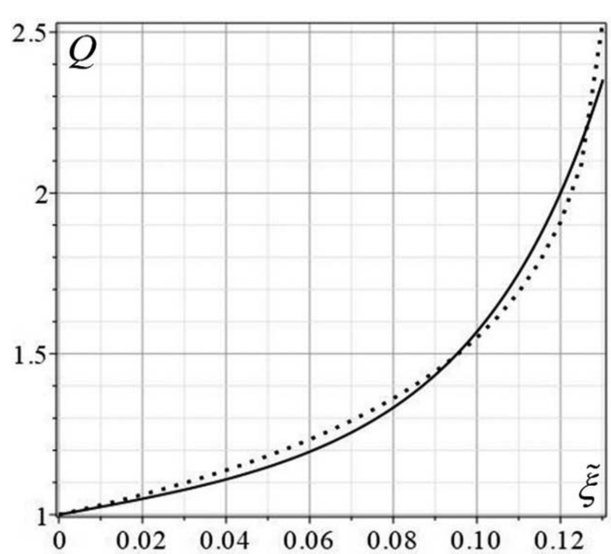

Figure 16 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.7$

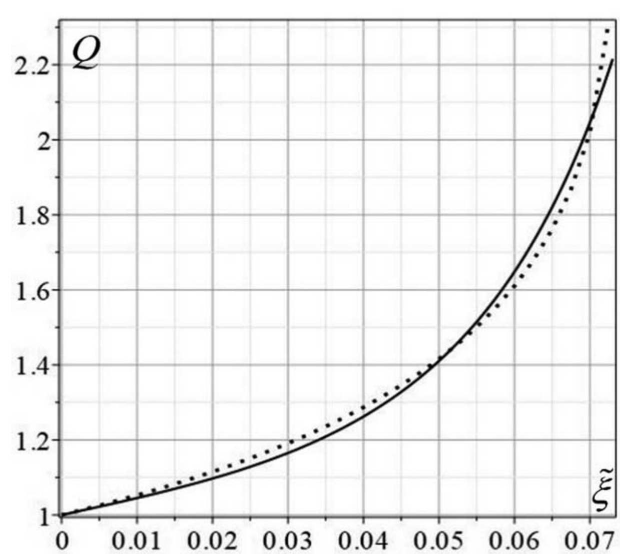

Figure 17 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.8$

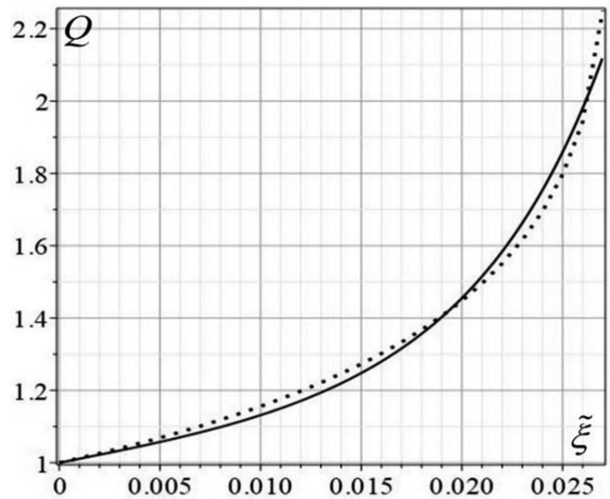

Figure 18 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.9$

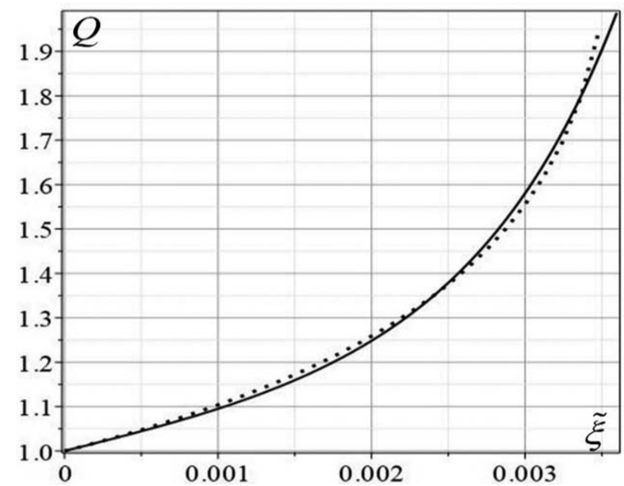

Figure 19 The value of Q from $\tilde{\xi}$ at $\varepsilon_0 = 0.975$

7 Conclusions

So, it has been proven that the non-Newtonian behavior of the lubricant significantly affects the value of the load factor $\Phi_P^\infty = \Phi_P^\infty(\varepsilon_0, \tilde{\xi})$. For the latter, a new, easy-to-use, adequate mathematical model was obtained, which is determined by formulas (2), (36) and representations (35), (37) – (39). Note that the existing representations for the load factor are usually tabular or graphical [2, 3, 4, 21], obtained for Newtonian lubricants and depend only on ε_0. The obtained mathematical model for the load factor, unlike the existing ones, in addition to the relative eccentricity ε_0, also takes into account the relative radial gap δ_0, the speed of rotation of the trunnion ω_0, as well as the viscosity gradient of lubricants $G_{\mu 0} = \xi\mu_0$. This is especially important for the application of the Sommerfeld criterion [2, 3, 21] when calculating the durability ship's sliding bearings in connection with the non-Newtonian behavior of ship's lubricants [15, 19].

The developed criterion (13), (15) makes it possible to determine at which values of the viscosity gradient $G_{\mu 0}$, the hydrodynamic pressure in the lubricating layer of the sliding pair will be finite, and the sliding bearing is in conditions of liquid friction. Calculations also show that the smaller the relative eccentricity ε_0, the greater the possible range of changes in values $G_{\mu 0}$ (see Table 1). In addition, the maximum possible value $G_{\mu 0}^*$ at a given relative eccentricity ε_0 increases with an increase in the relative radial clearance δ_0 and decreases with an increase in the angular velocity ω_0 (see Table 2).

The obtained results are of particular importance for monitoring the trouble-free operation of the sliding bearings of the ship's propulsion complex, which are constantly in extreme operating modes. This is because the viscosity gradient $G_{\mu 0}$ of marine lubricants during operation is constantly changing [16, 19, 20], either due to pollution or due to the use of various impurities. Therefore, in our opinion, during the long-term operation of the ship's propulsion system, it is necessary to conduct the following monitoring:

- determine, after a certain period of operation T_*, the viscosity gradient of marine lubricants $G_{\mu 0}$ and check the fulfillment of criterion (15);
- before using additives, determine the viscosity gradient $G_{\mu 0}$ of the mixture of marine lubricant and additives and check the fulfillment of criterion (15).

The indicated monitoring, in our opinion, should be performed within the shipping company in specialized laboratories, with the help of, for example, work methods [16, 17, 18]. The time interval can be determined experimentally for each type of engine, for different operating conditions.

The proposed approach can be applied to obtain adequate mathematical models for other important characteristics of the ship's sliding bearings, in particular, the coefficient of hydrodynamic friction [2, 3, 4, 5].

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